Nonlinear, Large Deformation Finite-Element Beam/Column Formulation for the Study of the Human Spine: Investigation of the Role of Muscle on Spine Stability

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Abstract: A nonlinear, large deformation beam/column formulation is used to model the behavior of the human spine under compressive

to the action of muscles: the tangent assumption and the secant assumption;

3. Use homogenization theory to determine the behavior of intervertebral disks from the knowledge of the response of soft tissues and geometry of the vertebrae; and
4. Reassess the frontal plane model for the lumbar spine with the presence of disks and vertebrae, gravity load, and nonlinear disk response for both the tangent assumption and the secant assumption.

The paper is organized as follows. In the next section, we introduce the beam-column formulation, derive the weak and strong forms of the governing equations, and present the nite-element discretization. In the "Homogeneous Column, Frontal Plane Model" section, after introducing the mechanical behavior of the column, we describe two models for the response of muscle, namely, the tangent and the secant models. We then reproduce the frontal plane model with muscle described by Patwardhan et al. 2001, compare results, and investigate differences between the two approaches. In the "Nonlinear Multiscale Spine Model with Disks" section, we describe a more realistic frontal plane model of the lumbar spine by including disks. We also derive the response of the column in bending from the hierarchical homogenization approach and discuss the implication of the new model. We nally discuss our results and potential research.

Three-Dimensional Reissner's Beam Theory for the Vertebral Column

Kinematics

Reissner's beam theory Kapania and Li 2003; Reissner 1981; Simo 1985; Simo and Vu-Quoc 1986; Vernerey et al. 2007 gives a mathematical description of a beam column based on six degrees of freedom: three displacements and three rotations. Let us consider a beam in the Cartesian coordinate system $e\,(= 1, 2, 3)$ for which the arc-length coordinate is given by the scalar. The initial con guration of the beam is de ned at point by the position vector $_0(\ )$ and three coordinate axes,

$$\delta W_{\text{int}} = \int A(\ )(\ \cdot \delta\ + \ \cdot \delta\ ) \qquad (11)$$

One can show that the virtual material strains have the following form:

$$\delta\ = (\mathbf{R} \cdot \mathbf{R}_0)\ \cdot [\ _{,}\ +\ _{,}\ ]$$

Eqs. 1175/F42 1 Tf30 04 /F12 1 Tf 0.78353 0 TD (3)Tj /F7- 1 368

$$\delta\ = (\mathbf{R} \cdot \mathbf{R}_0) \cdot \delta\ = \ _{,} +\ _{,}$$

$$\delta\ = (\mathbf{R} \cdot \mathbf{R}_0)\ \cdot [\ _{,}\ ] \quad \delta\ = (\mathbf{R} \cdot \mathbf{R}_0) \cdot \delta\ = $$

Se,a t M e

In the secant model, used by Patwardhan et al. 2001, we assume that the force $_I$ is such that the total resulting force at point $_I$ is directed along the vector $_I = (\ _I) \quad (\ _{I\,1})$ generated by two successive muscle attachments. If we introduce $_I$ as the vector perpendicular to $_I$, we obtain the condition

$$[\ _I + \ (\ _I)] \cdot \ _I = 0 \tag{31}$$

Using Eq. 22 in Eq. 26, we obtain

$$_I = \frac{(\ _I) \cdot \ _I}{_I \cdot \ _I} \tag{32}$$

Results

We rst propose to investigate the frontal plane model of the lumbar spine proposed by Patwardhan et al. 2001. The model consists of the lumbar portion of the column in a exed posture, with ten muscles ve on each side, pinned at the bottom end and subject to an increasing vertical load at the top end. The geometry of the column, muscle placement, dimensions, and boundary conditions are given in Fig. 3. The model described in Patwardhan et al. 2001 is based on a beam-column model Timoshenko and Gere 1961 that only accounts for bending. In order to reproduce the behavior in our more general model, we used the penalty method by introducing a high elastic modulus for tension/shear and torsion modes. Referring to Eq. 21, we take the elastic parameters to be $E = G = 10^9$. To reproduce the results of Patwardhan et al. 2001, the direction of muscle force $_I$ is kept constant during the simulation. Finally, similar to the study Patwardhan et al. 2001, the secant model for the muscles' action is used.

The simulation is performed with 200 nite elements, each with one quadrature point. The top load was increased in 100-N increments and the simulation was stopped when the reaction force on the base of the column reach5a8euwa52.3(1)-266.eNend.The investigate various features of the new model, including nonlinear effects from the motion of muscles during deformation, the tangent model for muscles' action, and the effects of including the response of the structure in shear and tension.

E et M e M t

Here, the direction of the muscle force is updated as the vertebral column deforms. The muscle forces, compressive load along the column, and the deformed con guration are plotted and compared with the model of Patwardhan et al. 2001 see Fig. 5.

Ta e t M e

The tangent model for muscles' action described in the section "Tangent Model" is now investigated. As in the previous studies, the top vertical load is incremented in 100-N steps until the total compressive stress in the bottom of the column reaches 1,200 N. Referring to Fig. 6, two observations can be made: First, the difference in reaction force along the spine between the two models is not signi cant, and second, large differences can be seen in the column de ection last gure and muscles' reactions top two node brick elements in the annulus and four node elements in the vertebrae. The discretization and the generated bending moment/ rotation angle curve are shown in Fig. 7.

The response of the disk in bending is then determined as follows: First, we make the assumption that bending of the unit vertebra-disk-vertebra is mainly due to the disk, due to the high stiffness of the bone material. The curvature κ of the disk can then be written as follows:

$$\kappa = \frac{\Delta\theta}{h} \approx \frac{\Delta\theta_{v\,v}}{h} \qquad (33)$$

where $\Delta\theta$ = angle difference between the top end and bottom end of the disk; $\Delta\theta_{v\,v}$ = angle difference between the top end and bottom end of the unit vertebra-disk-vertebra; and h = height of the disk. Finally, the moment-curvature curve for the intervertebral disk is determined and shown in Fig. 8. The bending stiffness of the disk is seen to increase with the level of bending strains.

Effect of the Nonlinear Disk Response

We investigate the nonlinear response of the disks and its effect on the structure. This computation is performed with both a linear and a nonlinear disk response.

Wt t M e At at

Let us rst examine the effect of the nonlinear disk response when muscles are not considered. The column is loaded at the top end, until the level of bending strain in the rst t

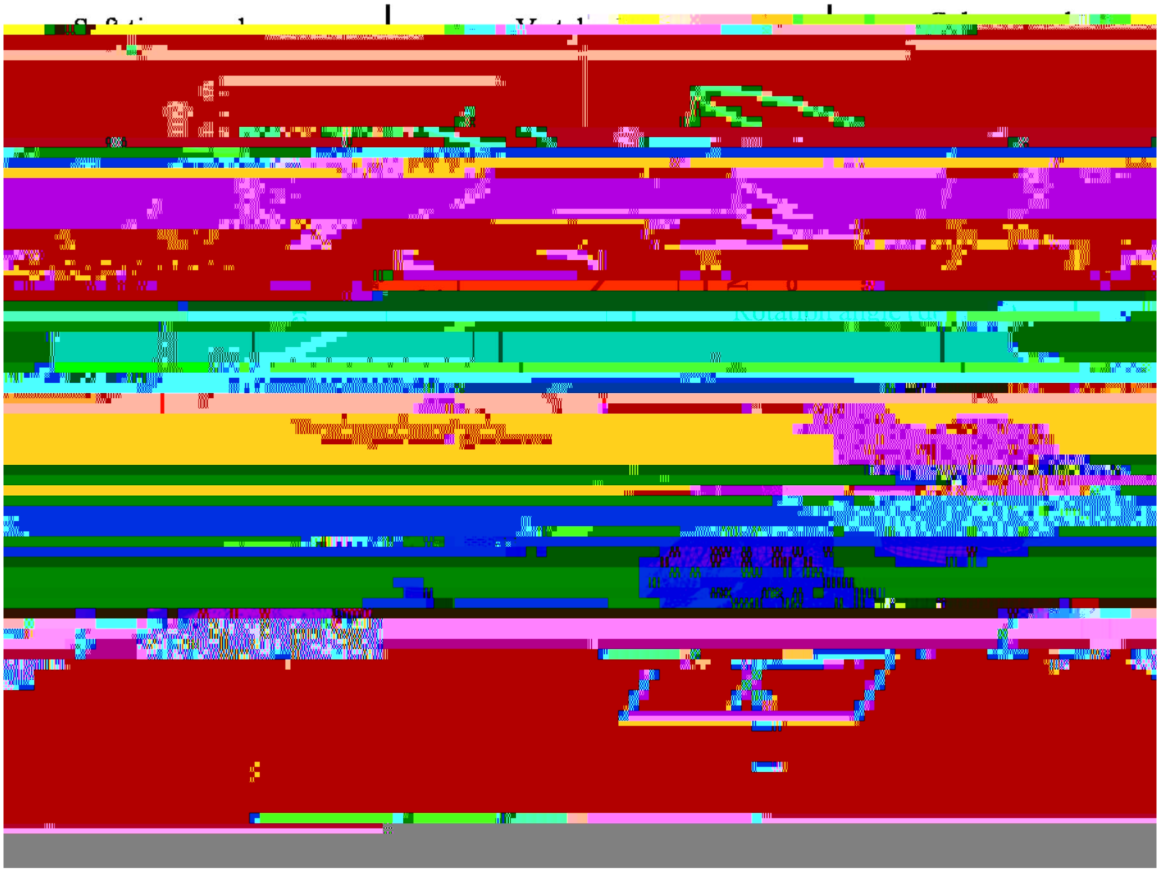

Fig. 7. Hierarchical multiscale framework to determine the response of a spine segment in bending; the soft tissue scale is represented by composite based model (Guo et al. 2007), the vertebral scale is represented by two cervical vertebrae and a disk; the mechanical response in bending is plotted; the column scale is represented by column elements with different properties in disks and vertebrae

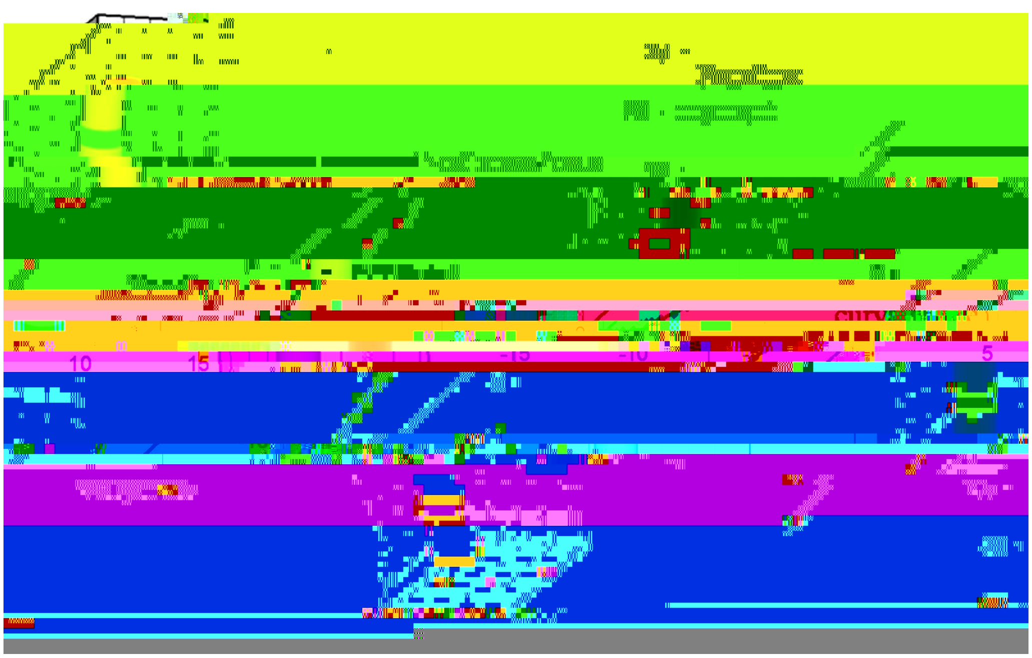

Fig. 8. 3D representation of the beam with disks and the computed moment-curvature curve for the disks; both linear and nonlinear responses are shown

J. Eng. Mech., 2010, 136(11): 1319-1328

distribution, muscle forces, and spine de ection. This emphasizes the fact that due to nonlinearities both from a geometrical and material point of view, small differences at small scales might have a signi cant impact at the scale of the entire structure. In particular, an alteration of the disk properties due to disease or damage might have a large impact on the overall spine. The need for the multiscale modeling of the human spine is therefore reinforced.

The presented model is exible and can be extended to describe a rich variety of spine mechanical response. Thus, the computational scheme can be enriched to include the arc-length algorithm such as proposed in Riks 1972 and allow detailed studies of spine instabilities in the form of buckling, for instance. Also, while the above model is formulated under static loading condition, it can be further extended to incorporate dynamical behavior of the spine in response to high-frequency excitations. For this, the contribution of the kinetic work has to be included in Eq. 17 ; this will ultimately add an inertial term and a time dependency in the governing equation of the spine Reddy 1999 . In addition, experimental analysis of spine mechanics in dynamic conditions can be performed to derive assumptions regarding the dynamical response of muscles. Additional future research involves developing a constitutive response for the disks accounting for the six degrees of freedom, namely, tens s disks and ligaments in terms of their microstructure, and ultimately the behavior of the entire spine.

Acknowledgments

The writers would like to thank Medtronic-Sofamor Danek for research support.

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