increasing height results from the fact that for tall dots the hole states are of $_1$ (S-like) symmetry. The Bloch part of the electron state is of $_1$ symmetry in zinc-blende structure. The electron states in the following are therefore, including spin, twofold (Kramer's) degenerate. Furthermore, the contribution of the valence $|Z\rangle$ band in the hole states is neglected since it is pushed down in energy through the strong confinement in z direction (for flat dots). We will proceed in steps from the idealized cylindrical symmetry neglecting at first the spin-orbit interaction [Fig. 3(a)], to the full atomistic symmetry ($C_{2\text{v}}$) in the presence of the spin-orbit interaction [Fig. 3(d)]. We will show how the observed FS is the result of the atomistic symmetry in presence of the spin-orbit interaction.

Cylindrical symmetry, no spin-orbit interaction [Figs. 3(a) and 4(a)]. In this case, the hole states are eigenfunctions of the angular momentum $l=1$ as depicted in Fig. 4(a). The spin parts of the wave functions are written as $|\uparrow\rangle$ and $|\downarrow\rangle$. Due to the equivalence of the wave functions $|X\rangle$ and $|Y\rangle$ in cylindrical symmetry, the four hole states are degenerate. The resulting eight exciton states (two electrons, four holes) are split by the exchange interaction K (singlet-triplet splitting) into two $S=0$ and six $S=1$ states.

$C_{2\text{v}}$ symmetry, no spin-orbit interaction [Figs. 3(b) and 4(b)]. The spin-independent $C_{2\text{v}}$ potential does not have the ability to mix spins. However, it will mix the orbital parts of isospin hole states creating the eigenstates given in Fig. 4b, where the $C_{2\text{v}}$ point-group notation[17] has been used. We obtain two pairs of eigenfunctions whose orbital parts belong to the $_2$ and $_4$ representations and spin parts to the $_5$ representation. The splitting of these two pairs is due to the nonequivalence of the $|{}_{2v}\rangle$ and $|{}_{4v}\rangle$ Bloch functions (atomistic asymmetry), reflected in the atomistic asymmetry parameter $= {}_{2v}|H_{C_{2\text{v}}}|{}_{2v}\rangle - {}_{4v}|H_{C_{2\text{v}}}|{}_{4v}\rangle$, which is characteristic of the $C_{2\text{v}}$ potential. The previously fourfold degenerate hole states split into two by 2 . Consequently, the *exciton* states are split by the atomistic asymmetry 2 and further split into singlet and triplet by the exchange term K [Fig. 3(b)].

Cylindrical symmetry, with spin-orbit interaction [Figs. 3(c) and 4(c)]. The spin-orbit interaction splits the hole states with respect to their total angular momentum J. Thus, the $J_z=3/2$ hole states $a\uparrow$ and $b\downarrow$ will split by $_0$ from the $J_z=1/2$ states $a\downarrow$ and $b\uparrow$ [see Fig. 4(c)]. Considering only the first two hole states ($a\uparrow$, $b\downarrow$) and the electron states ($e\downarrow$, $e\uparrow$), the exchange Hamiltonian in the basis of the four excitons $(a\uparrow e\uparrow)$, $(a\uparrow e\downarrow)$, $(b\downarrow$ $b\downarrow e\downarrow$