(2019) **8**, 289 312
o :10.1093/ m / 010
A 4 A44 4 o o 3 J 2018

Ensemble-based estimates of eigenvector error for empirical covariance matrices

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[4 o 27 m r 2016 r o 28 r 2018 44 o 6 A r 2018]

o r 4 m r 4 r fi m o for 4 of 4o o m 4, 4 o og 4

1. Introduction

4 r ro r of 4o r 4 m r 4 r 4 r o 4 m m 4 , ro 4 (, 1991 r o , 2003 , 2009 o & o , 2012) ro 4or r-

o . or m, o o from (1.5) 4 g g o

E[ ] +

Assumption 2.2 m o ro r o (,) of f r g g , + λ λ 1 , ro 4 g λ g fo o g g r g r m for

$$(\; , \;) \quad \frac{3^7 [\, \rho(\lambda\,)]^5}{32\pi^3} [\; , \langle$$

2.3 … 2 …

… o (2.2) g … m o 4 m for …

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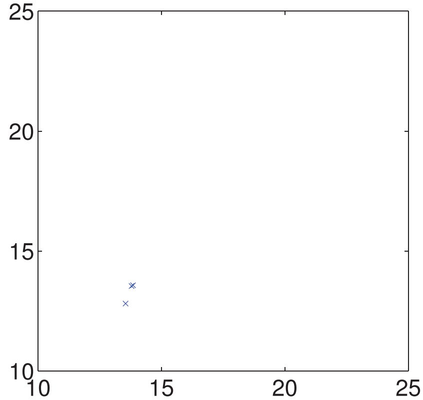
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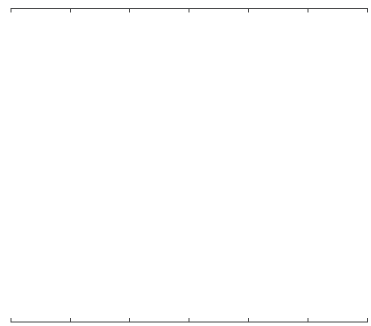

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A. Derivation of main result 1

A. , ro m (1.1) m of r - g o r g g . B o g o, o o k o g of , m g r o of g . g g mm o o o r o

$$+ \quad , \tag{A.1}$$

$$+ \sum_{+1}^{1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.2}$$

$$+ \sum_{+ \, 1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.3}$$

r m r 4 r m o 4 r - g o r m om o r . T k g o 44o , o r 4 g r r

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \quad \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.4}$$

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \quad \sum_{+ \, 2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.5}$$

rg o r of (A.4) (A.5) r 4o r g r - g o r m mm o . r 4 r, o ro m o r o o r g g 0 , , (A. m o 2.2 4o rg 4 , 4 o , r + (1/)), m o 4 m

$$\frac{\lambda\lambda_{1}}{(\lambda \quad \lambda_{1})^2} \qquad \frac{\lambda^2}{(\ )^2}. \tag{A.9}$$

o r or o o mm o , ro m g m g , 4 r $\rho(\lambda)$ of orm m r 4 4o g m r of g . or r 4 , 4o r 4 of - mm r 4 4o r 4 m r 4 , g g λ for 1, . . . , . for 4 m r m r 4 4 r

$$\rho(\lambda) + \ ^{1}\sum \delta(\lambda), \tag{A.10}$$

r $\delta(\lambda)$ 4 fi 4 o λ R. m 4o r 4 m r 4 r r from m 4 $\rho(\lambda)$ k 4o rg , m g

$$\int \rho(\lambda)(\lambda)\ \lambda \qquad \int \rho(\lambda)(\lambda)\ \lambda \tag{A.11}$$

for 4o o fi 4 o (λ). m $\rho(\lambda)$ 4o , , 4om 4 or (o (ρ)) ff r o (ρ). or o o 4o 4 , m $(\rho) + (\alpha, \beta)$ for om α, β R, o g o r 4 m of gr o (A.11) (α, β). o r, r 4 o o of 4 r . g r r g mm o (A.4) (A.5) gr o of fi 4 o

$$\lambda(\lambda) + \frac{\lambda\lambda}{(\lambda \quad \lambda)^2} \tag{A.12}$$

ro m r $\rho(\lambda)$ g (A.10),

$$\frac{1}{\ }\sum_{+1}^{2} \frac{\lambda\lambda}{(\lambda \quad \lambda)^2} + \int_{\alpha}^{\lambda\ 1} \rho(\lambda)_{\lambda}(\lambda)\ \lambda, \tag{A.13}$$

$$\frac{1}{\ }\sum_{+\ 2\lambda}$$

o r o r g of (A.15) 4 ε. g o $\rho(\lambda)$ (λ) r ff r for λ r (ρ) ‖ λ, m m gr o r , r g m r or om or r , o o

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \; + \; \lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \quad \lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{\rho(\lambda) \quad \lambda\rho\,(\lambda)}{\lambda \quad \lambda} \quad \lambda. \tag{A.16}$$

r m r g of (A.16) ε 0 m o 4 m

$$\lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \qquad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.17}$$

4o m o r g of (A.16) o

$$\left|\lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{[\rho(\lambda) \quad \lambda\rho\,(\lambda)]}{\lambda \quad \lambda} \quad \lambda\right| \leqslant \lambda \left[\quad \rho(\lambda) \quad \lambda\rho\,(\lambda) \right]_{(\alpha,\lambda \quad \varepsilon]} \int_{\alpha}^{\lambda \quad \varepsilon} \frac{1}{\lambda \quad \lambda} \quad \lambda$$

$$+ \lambda \left[\quad \rho(\lambda) \quad \lambda\rho\,(\lambda) \right]_{(\alpha,\lambda \quad \varepsilon]} \left(\frac{\lambda \quad \alpha}{\varepsilon}\right). \tag{A.18}$$

fo o 4o m r g of (A.16) 4 g ($(1/\varepsilon)$) om m ε 0 r m, 4 $(1/\varepsilon)$. 4om (A.17) (A.18) o o ε 0 m o 4 m

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \qquad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.19}$$

o 4 r $\rho\,(\lambda)$ o , r g for r o r gr o f of (A.18) o o om : o 4o g λ, r $\rho\,(\lambda)$ o 4o om g r m λ ε. r gr 4o rg o ro o (A.11) r 4o o g (A.18), m g gr m (A.16) ($(1/\varepsilon)$), ro $\rho(\lambda)$ ff r m g o r oo 4o g λ. m g o r for r g of (A.13) 4o r g fo o g :

$$\int_{\alpha}^{\lambda \quad 1} (\lambda)\rho(\lambda) \quad \lambda + \int_{\alpha}^{\lambda} (\lambda)\rho(\lambda) \quad \lambda \quad \int_{\alpha}^{\lambda} (\lambda)\left[\rho(\lambda) \quad \rho(\lambda)\right] \quad \lambda$$

mm r , 4om (A.21), (A.22) (A.17

To o m for r o of (), ff r (B.6) r 4 o oo

$$\partial_{-}() + \frac{\partial}{\partial} \int_{0()} \int_{(, ,)} (,) \qquad \text{(B.7)}$$

$$+ \frac{\partial^{0}}{\partial}() \int_{(, ^{0}())} (, ^{0}()) \int_{0()} \frac{\partial}{\partial} \left[\int_{(, ,)} (,) \right]$$

$$+ \int_{0()} ((, ,),) \frac{\partial (, ,)}{\partial} .$$

o o r o, r m 4o 4 (, ,) , m $^{0}()$ (,) o .

C. Derivation of main result 3

r 44or g o (), g (B.7), r 4 o m o 4 r o for () m , . m g (2.2), o = (,) rg /or r m, rg m o 4 4o r o 4o r o of m ro or o o $_2$ ^{2},

$$\frac{\lambda^2}{( )^2} \quad \frac{\lambda^2}{( )^2}. \qquad \text{(.1)}$$

4 ,

$$^{0}() + \frac{\lambda}{}, \qquad \text{(.2)}$$

$$(, ,) + \frac{\lambda}{[( )^2 \quad \lambda^2]^{1/2}}, \qquad \text{(.3)}$$

$$\frac{\partial}{\partial} ((, ,)) + \frac{\lambda( )^3}{2[( )^2 \quad \lambda^2]^{3/2}} + \frac{1}{2\lambda^2}[(, ,)]^3. \qquad \text{(.4)}$$

g o (B.7) ro g rg m for , . . . , (, ,) / ,

$$() + \int_{\lambda^2/} \left(\frac{3^7 [\rho(\lambda)]^5}{32\pi^3} (, ,) \quad \frac{[3\rho(\lambda)]^2}{4\pi} [( )^2 ( )^2] \right) \left(\frac{( )^3}{2\lambda^2} \right)$$

$$+ \frac{3^7 [\rho(\lambda)]^5}{32\pi^3} \frac{1}{2\lambda^2} \int_{\lambda^2/} \left(\left(( )^5 \quad ( )^2 ( )^4 \right) \quad \frac{[3\rho(\lambda)]^2}{4\pi} [( )^2 ( )^2] \right) .$$

4 g of r $+ ( )^2 \quad \lambda^2$ r form o

$$() + \frac{3^7 [\rho(\lambda)]^5}{32\pi^3} \frac{\lambda^2}{4} \quad ^{7/2} (), \qquad \text{(.5)}$$

o , 4 g for gr $_1(\,)$, $_2(\,)$ $_3(\,)$. B g g (.4), r o for r g (0, $_1(\,)$]

$$(\quad \lambda^2)^{5/2}(\ ^{1/2} \quad \lambda) \leqslant (\ _1 \quad \lambda^2)^{5/2}(\ _1^{1/2} \quad \lambda). \qquad (\ .7)$$

fo o

fo o

$$\left(1 \quad \frac{\lambda^2}{}\right)$$

$$\left(\ ^{/2}\right.$$

r 4o > 0 $\lambda^2/$ > 0 o

$$\varphi( ) + \frac{[3 \rho(\lambda)]^2}{4\pi}(\lambda^2)\Big[1 (\lambda^2/) (\lambda^2/)^{1/2}\Big]$$

$$\geqslant \frac{[3 \rho(\lambda)]^2}{4\pi} . \quad (17)$$

4 g of r $+ \frac{[3 \rho(\lambda)]^2}{4\pi}$ o o

$$_3( ) \leqslant 8 \left(\frac{[3 \rho(\lambda)]^2}{4\pi}\right)^{3/2} \int_{\frac{[3 \rho(\lambda)]^2}{4\pi} \, _2( )/}^{} \quad ^{1/2} . \quad (18)$$

rom (3), o r m of gr o 4o rg $\frac{[3 \rho(\lambda)]^2}{4\pi} \, _2( )/$ 1 gr (18) 4o rg o 4o . r for , $_3( )$ o m 4 g $^{3/2}$. o o $_2( )$ 4 g $(^{3/2})$ (o o o o r m , o r m 4 $^{3/2}$). o 4 of r o of $_1$ $_2$, g $\varphi( )$ r m m m $+ \lambda^2 \bullet (_1, _2)$,

$$\varphi(\lambda^2)/ \leqslant \varphi( )/ \leqslant 1 \quad (19)$$

for $\bullet (_1, _2)$. g o (6), $_2( )$

$$( ) \; ^1 \leqslant \, _2( ) \leqslant ( ) \; ^{\varphi(\lambda^2)/}, \quad (20)$$

r .

$$( ) \int_{_1( )}^{_2( )} \left(1 \frac{\lambda^2}{}\right)^{5/2} (^{1/2} \lambda) . \quad (21)$$

g m o 4 ro m o for $_1( )$ $_2( )$ g (2) (3), gr (21) g of r m 4 (g r [( ), ,1]' 4o m) o o m o 4 o r,

$$( ) \frac{2^4 \pi^{3/2}}{3^4 [\rho(\lambda)]^3} \, ^{3/2}. \quad (22)$$

r mor , 4o m $\varphi(\lambda^2)/$ 0 (20) o o m o 4

$$( ) \; ^1 \leqslant \, _2( ) \leqslant ( ). \quad (23)$$

4o m (23) (15) (18) o o rg 4 g $( ) + \left(\frac{^{3/2}}{^3}\right)$.