Program in Applied Mathematics
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION
January 2014

Notice: Do four of the following five problems. Place an X on the line opposite the number of the problem that you are NOT submitting for grading. Please do not write your name anywhere on this exam. You will be identified only by your student number, given below and on each page submitted for grading. Show all relevant work.

1. ____
2. ____
3. ____
4. ____
5. ____
Total ____

Student Number _____

1. Consider $U \sim Uniform(0, 1)$ and let R be a continuous random variable with probability density function $f(r) = re^{-r^2/2}$, for $r > 0$. Define:

$$X := a + bR\cos(2\pi U)$$
$$Y := c + dR\sin(2\pi U)$$

where

3. Consider i.i.d. random variables $X_1, \ldots, X_n$ generated from the *Maxwell density*:

$$f(x) = \sqrt{\frac{2}{\pi}} \frac{x^2}{\sigma^3} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}; \qquad x > 0; \quad \sigma > 0.$$

Note this family satisfies the "nice" regularity properties that are useful for examining maximum likelihood estimators. This density describes the distribution of speeds of molecules in thermal equilibrium.