

# On the finiteness in the deformed Hamiltonian mean-field model

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The deformed Hamiltonian mean-field model is a system of particles interacting via a potential that is a deformation of the Coulomb potential. The system is studied in the limit of large number of particles  $N$  and large deformation parameter  $\alpha$ . The system is shown to exhibit a transition from a state of finite energy to a state of infinite energy as  $\alpha$  increases. The transition is characterized by a change in the behavior of the energy per particle  $\epsilon$  as a function of  $\alpha$ . For  $\alpha < \alpha_c$ , the energy per particle is finite and increases with  $\alpha$ . For  $\alpha > \alpha_c$ , the energy per particle diverges as  $\alpha$  increases. The critical value  $\alpha_c$  is found to be approximately 1.2. The transition is shown to be a first-order phase transition. The system is shown to exhibit a rich phase diagram in the  $(\alpha, N)$  plane. The phase diagram is shown to have a region of finite energy and a region of infinite energy. The boundary between the two regions is shown to be a curve that separates the two regions. The system is shown to exhibit a rich phase diagram in the  $(\alpha, N)$  plane. The phase diagram is shown to have a region of finite energy and a region of infinite energy. The boundary between the two regions is shown to be a curve that separates the two regions.

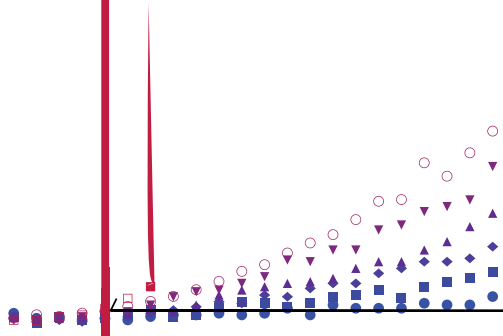
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## I. INTRODUCTION

The deformed Hamiltonian mean-field model is a system of particles interacting via a potential that is a deformation of the Coulomb potential. The system is studied in the limit of large number of particles  $N$  and large deformation parameter  $\alpha$ . The system is shown to exhibit a transition from a state of finite energy to a state of infinite energy as  $\alpha$  increases. The transition is characterized by a change in the behavior of the energy per particle  $\epsilon$  as a function of  $\alpha$ . For  $\alpha < \alpha_c$ , the energy per particle is finite and increases with  $\alpha$ . For  $\alpha > \alpha_c$ , the energy per particle diverges as  $\alpha$  increases. The critical value  $\alpha_c$  is found to be approximately 1.2. The transition is shown to be a first-order phase transition. The system is shown to exhibit a rich phase diagram in the  $(\alpha, N)$  plane. The phase diagram is shown to have a region of finite energy and a region of infinite energy. The boundary between the two regions is shown to be a curve that separates the two regions.

The evolution of the density is given by the continuity equation





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