PHYSICAL REVIEW E **84**, 036208 (2011)

Cluster synchrony in systems of coupled phase oscillators with higher-order coupling

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(Received 7 July 2011; published 16 September 2011)

We study the phenomenon of cluster synchrony that occurs in ensembles of coupled phase oscillators when higher-order modes dominate the coupling between oscillators. For the first time, we develop a complete analytic description of the dynamics in the limit of a large number of oscillators and use it to quantify the degree of cluster synchrony, cluster asymmetry, and switching. We use a variation of the recent dimensionality-reduction technique of Ott and Antonsen [Chaos **18**, 037113 (2008)] and find an analytic description of the degree of cluster synchrony valid on a globally attracting manifold. Shaped by this manifold, there is an infinite family of steady-state distributions of oscillators, resulting in a high degree of multistability in the cluster asymmetry. We also show how through external forcing the degree of asymmetry can be controlled, and suggest that systems displaying cluster synchrony can be used to encode and store data.

DOI: 10.1103/PhysRevE.84.036208 PACS number(s): 05.45.Xt, 05.90.+m

I. INTRODUCTION

Large systems of coupled oscillators occur in many re-

Cluster synchrony has been studied in many contexts, for example, in networks of phase oscillators with [

with phase θ and natural frequency ω at time t. Since oscillators are conserved, f must satisfy the continuity equation $\partial_t f + \partial_\theta(f\dot{\theta}) = 0$, giving

$$\partial_t f + \partial_\theta \left\{ f \left[\omega + \frac{K}{2i}(r_2 e^{-2i\theta} - r_2^* e^{2i\theta}) \right] \right\} = 0. \tag{6}$$

To analyze Eq. (6), we find it convenient to define the symmetric and antisymmetric parts of f, f_s, and f_a, as

$$f_{s/a}(\theta,\omega,t) = [f(\theta,\omega,t) \pm f(\theta+\pi,\omega,t)]/2, \tag{7}$$

where f_s and f_a are symmetric and antisymmetric with respect to translation by π, respectively, in the sense that $f_s(\theta+\pi,\omega,t) = f_s(\theta,\omega,t)$ and $f_a(\theta+\pi,\omega,t) = -f_a(\theta,\omega,t)$. We note that f is a solution of Eq. (6) if $f = f_s + f_a$ and f_s and f_a are both solutions of Eq. (6). Thus, we can study separately the symmetric and antisymmetric dynamics of solutions f.

A. Symmetric dynamics

while the antisymmetric density f_a remains the same. Thus, the only change in $|r_1$

Problems that remain open include generalizations such as the presence of noise and coupling functions with two or more harmonics. Thus far the work of Ott and Antonsen [19

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