PDE Preliminary Examination: Spring 2012

Name: ____

There are 5 problems. Each problem is worth 25 points. You are required to do 4 of them. Please indicate which 4 you choose–Note: Only 4 problems will be graded. A sheet of convenient formulae is provided.

#	possible	score
1	25	
2	25	
3	25	
4	25	
5	25	
Total	100	

$0 < x < 1;\ t > 0;\ a > 0$

$0 < x < 1;$ (1)

$0;$

$0:$

principle.

$u(x;t) = \int_0^1 g(x;y;t) f(y)dy$. In the case that $F(x;t)$

$x;z;t \quad s)g(z;y;s)dz$ for $t > s > 0$.

ess of solutions to (1).

---

2. Fourier Series.

(a) Show explicitly a Fourier series and an open interval $S = (a;b)$ such that the series converges pointwise in S but does not converge uniformly in S.

(b) State the Weirstrass approximation theorem with any assumptions necessary.

(c) Suppose $f(x)$ is a continuous 2 periodic function. Prove that

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(2\ n\ ) = \frac{1}{2} \int_0^2 f(x)dx \tag{2}$$

for any irrational .

---

3. Method of Characteristics.

Solve $(t^2 \quad 1)\ u_t(t;x) \quad x u_x(t;x) = u$, with the initial condition $u(0;x) = e^x$. (Solve all problems in terms of the original VARIABLES!)

TURN OVER

4. Wave equation.

Consider the forced wave equation

$$u_{tt} = u_{xx} \quad e^{x}; \qquad t > 0; \qquad 0 \quad x \quad L: \tag{3}$$

with initial conditions $u(x;0) = g(x); u_t(x;0) = 0$; and boundary conditions $u(0;t) = u(L;t) = 0$.

(a) Find a formal solution in terms of the function g.

(b) Find conditions on g that guarantee that the expression you found in (a) is a solution of the system.

---

5. Laplace's Equation

Let $B = B_a(0) \quad R^2; a > 0$. Consider the f2.531.8Td27.5024(e)3.56148(r)341.911(t)0.649399(h)1.95024(e)327.689(f)1.94207(2.1671(u