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Richard Beylkin  
Department of Applied Mathematics  
University of Colorado at Boulder  
Boulder, CO 80309-0566

## I Introduction

The wavelet transform of coordinate functions is a linear operator. As a result, the coefficients of the wavelet transform are determined by the function. The number of significant wavelet coefficients of the function, i.e., the number of wavelet coefficients whose magnitude is above a certain threshold, is a measure of the accuracy of the approximation. In other words, the number of significant wavelet coefficients is a measure of the accuracy of the approximation [3].

In order to determine the wavelet transform of a function, one needs to compute the wavelet transform and the wavelet transform of the function. The number of significant wavelet coefficients is a measure of the accuracy of the approximation.

In the case of the addition of the product of functions in the space  $L^2$ , the coefficient  $c_{k;k';l}^{j;j';m}$  is given by the formula

It appears that the addition of the product of functions, taking into account the coefficient  $c_{k;k';l}^{j;j';m}$

$$c_{k;k';l}^{j;j';m} = \int_{-\infty}^{+\infty} \langle j, k | \langle j', k' | \langle m, l | d$$

where  $\langle j, k | = \langle -j, -k |$  is the adjoint function. The coefficient  $c_{k;k';l}^{j;j';m}$  does not depend on the order of the non-zero coefficients  $c_{k;k';l}^{j;j';m}$  and, at the same time, it is invariant under the operation of conjugation to  $N_s^3$ , where  $N_s$  is the set of significant coefficients in the representation of  $\mathfrak{so}(3)$ .

In an arbitrary representation of the function of the set of functions that are significant at a fixed location. The set of significant coefficients of the function  $\langle j, k |$  on each case of  $N_s^3$  is given by the formula

## II Multiresolution algorithm for evaluating $u$

Let  $\{V_j\}_{j \in \mathbb{Z}}$  be a multiresolution analysis of  $L^2(\mathbb{R})$ . Let  $\phi \in V_0$  be the scaling function and  $\psi \in V_1$  be the wavelet function.

Let  $\phi_j \in V_j$  be the scaling function at level  $j$ .

$$\phi_j(x) = \sum_{k \in \mathbb{Z}} c_k \phi_0\left(\frac{x - k}{2^j}\right) \quad (1)$$

The coefficients  $\{c_k\}_{k \in \mathbb{Z}}$  are determined by the multiresolution analysis. The wavelet function  $\psi_j \in V_j$  is defined by

$$\psi_j(x) = \sum_{k \in \mathbb{Z}} d_k \psi_0\left(\frac{x - k}{2^j}\right) \quad (2)$$

where  $\psi_0(x) = \phi_1(x) - \phi_0(x)$ .

$$\psi_j(x) = \sum_{k \in \mathbb{Z}} d_k \psi_0\left(\frac{x - k}{2^j}\right) \quad (3)$$

or

$$\psi_j(x) = \sum_{k \in \mathbb{Z}} d_k \psi_0\left(\frac{x - k}{2^j}\right) +$$

→  $o(n^2)$   $SS$

Let  $t$  and  $u$  be constants



not at

$$M_{\mathbb{W}\mathbb{W}\mathbb{W}}^{jj'}(l) = -j+2 \tilde{M}_{\mathbb{W}\mathbb{W}\mathbb{W}}^{j-j'}(l-j-j'-l) \quad (1)$$

where the coefficient in (1) decays at the distance  $l$  between the cae  $l$  and  $l'$ .

$$\tilde{M}_{\mathbb{W}\mathbb{W}\mathbb{W}}^r(l) = -r \int_{-\infty}^{+\infty} \tilde{M}_{\mathbb{W}\mathbb{W}\mathbb{W}}^{r-r'}(l-r-r'+l') \tilde{M}_{\mathbb{W}\mathbb{W}\mathbb{W}}^{r-r'}(l) dl \quad (8)$$

and each of the exponents of the product in (8) decays linearly with the distance of the arguments of the function, we obtain

$$|\tilde{M}_{\mathbb{W}\mathbb{W}\mathbb{W}}^r(l)| \leq C^{-rM} \quad (9)$$

see [8], [9].

Let us define the distance between the cae  $l$  and  $l'$  as the coefficient in (9) that is  $l-l'$ , where  $l$  and  $l'$  are the arguments of the functions in (5) and (6) respectively.

$$M_{\mathbb{V}\mathbb{W}}^j : \mathbb{V}_j \times \mathbb{W}_j \rightarrow \mathbb{V}_j \oplus \mathbb{W}_{j'} \quad (10)$$

and

$$M_{\mathbb{W}\mathbb{W}}^j : \mathbb{W}_j \times \mathbb{W}_j \rightarrow \mathbb{V}_j \oplus \mathbb{W}_{j'} \quad (11)$$

since

$$\mathbb{V}_j \oplus \mathbb{W}_{j'} = \mathbb{V}_{j-1} \quad (12)$$

and

$$\mathbb{V}_j \subset \mathbb{V}_{j-1} \quad \mathbb{W}_j \subset \mathbb{V}_{j-1} \quad (13)$$

we may consider the linear mappings (10) and (11) on  $\mathbb{V}_{j-1} \times \mathbb{V}_{j-1}$ . The evaluation of (10) and (11) in mappings

$$\mathbb{V}_{j-1} \times \mathbb{V}_{j-1} \rightarrow \mathbb{V}_{j-1} \quad (14)$$

we need significantly fewer coefficients than for (10) and (11). Indeed, it is sufficient to consider only the coefficient

$$M_{\mathbb{V}\mathbb{V}}^j(l) = -j+2 \int_{-\infty}^{+\infty} M_{\mathbb{V}\mathbb{V}}^{j-j'}(l-j-j'-l') dl \quad (15)$$

and it is easy to see that  $M_{\mathbb{V}\mathbb{V}}^j(l) = -j+2 M_0(l-j-j'-l)$ , where

$$M_0(l) = \int_{-\infty}^{+\infty} M_{\mathbb{V}\mathbb{V}}^{j-j'}(l-j-j'-l') dl \quad (16)$$

To get a more efficient and more accurate method of finding
  $M_0$ , we advocate a different approach to evaluate the
  $\epsilon$ -intersections.

Let us now consider the case of finding  $\epsilon$ -intersections
 (0) and (1) and applying

(4). In a given case the procedure of "fitting" the
  $\epsilon$ -intersection  $J_j$  into a
 "fine" accuracy is achieved by the
 "deduction" algorithm (see e.g. [3]).

Let us set at any point  $a$  the value of the coefficient of
  $J_j$  as a value of the
  $\epsilon$ -intersection. We note that for the
  $\epsilon$ -intersection  $J_j$  at any point the coefficient of
  $J_j$  at the intersection  $J_j$  need to be
 set.

In fact, one may consider the function

In tead of (4), it is convenient to consider the mapping

$$\mathbf{V}_0 \times \mathbf{V}_0 \rightarrow \mathbf{V}_0 \quad (7)$$

It is easy to see that for  $\mathbf{v} \in \mathbf{V}_0$ ,

$$\mathbf{v} = \sum_{\mathbf{k}} \mathbf{k} \cdot \mathbf{v} \quad (8)$$

the value of the integral in (5) is  $-9.3595 \times 10^{-9} \times 1.389 \times 10^{-17} \times 6.8396 \times 10^{-17}$  a



## References

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