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### abstract

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$$S_\infty(r) = \frac{h}{\Gamma(\cdot)} \sum_{n \in \mathbb{Z}} e^{(t_0+nh)} e^{-e^{t_0+nh}r}. \tag{13}$$

$$\sum_{n \neq 0} \frac{|(t_0+nh) - i\frac{\pi}{h}|}{\Gamma(\cdot)} < \dots \tag{14}$$

3. Given  $\epsilon > 0$  and  $0 < \delta \leq 1$ , for any step size  $h$  such that

$$h \leq \frac{2}{3 + (\delta - 1)^{-1} + \delta^{-1}}, \tag{15}$$

and any  $t_0 \in \mathbb{R}$  we have

$$\frac{|r^\delta - S_\infty(r)|}{r^\delta} \leq \epsilon, \text{ for all } r > 0, \tag{16}$$

where  $S_\infty$  is given in (13).

$$\dots \tag{16}$$

4. For all  $r > 0$ ,

$$S_F(r) < S_\infty(r) < (1 + \epsilon)r^{-\epsilon}.$$

12. 4  $S_F(r)$  on the whole positive axis, 9.

(16).  $f$  (10)  $r$

$S_F$

5. For any  $\epsilon > 0$ ,  $\delta > 0$ , and  $1 - \epsilon > \delta$ , 9.7304 0 0 9.7304 303, ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.

Let  $t_M \leq \dots$   $r \in [0, 1]$   $y_0 = (r^{-1}) \dots$   $M$   $T_M(r)$

$$T_M(t) \leq \frac{r}{(\cdot)} \int_{-\infty}^{t_M} e^{-re^y + y} dy \leq \frac{1}{(\cdot)} \int_{-\infty}^{t_M} e^{-e^y + y} dy \tag{27}$$

$$= \frac{1}{(\cdot)} \int_0^{e^{t_M}} e^{-s} s^{-1} ds = 1 - \frac{(\cdot, e^{t_M})}{(\cdot)}, \tag{28}$$

$$(\cdot, x) = \int_x^{\infty} e^{-s} s^{-1} ds$$

$t_N \geq \dots$   $N$   $(-1)$   $t_M$   $(29)$

$$T^N(t) \leq \frac{r}{(\cdot)} \int_{t_N}^{\infty} e^{-re^y + y} dy = \frac{1}{(\cdot)} \int_{re^{t_N}}^{\infty} e^{-s} s^{-1} ds,$$

$$r \in [0, 1] \dots T^N(r) \leq \frac{(\cdot, e^{t_N})}{(\cdot)}. \tag{30}$$

$$\lim_{x \rightarrow 0} \frac{(\cdot, x)}{(\cdot)} = 1, \quad \lim_{x \rightarrow \infty} \frac{(\cdot, x)}{(\cdot)} = 0,$$

$$1 - \frac{(\cdot, e^{t_*})}{(\cdot)} = \dots, \tag{31}$$

$$\frac{(\cdot, e^t)}{(\cdot)} = \dots \tag{32}$$

$t_0, \dots, (t_* - t_0)/h, \dots, M_*, N_*, S_F, \dots, t_*, t^*, h$

7. For all  $\dots > 0, \dots > 0$  and  $1/e \geq \dots > 0$ , the solution  $t_*$  of (31) does not depend on  $\dots$  and satisfies 
$$t_* \geq \frac{(1 + \dots)}{\dots} = \frac{1}{\dots} + \dots (1 + \dots)^{\frac{1}{\dots}}. \tag{33}$$

The solution  $t^*$  of (32) has a weak dependence on  $\dots$  and satisfies

$$t^* \leq \dots^{-1} + \dots^{-1} + \frac{1}{2}. \tag{34}$$

$t_*, t^*, \dots, S_F, \dots, (33) (34), \dots, A, \dots, A.2. \dots, (9) (10) \dots, -\infty, \dots, +\infty, \dots, e^{hn} \dots, (21) \dots, 3, \dots, n), A, \dots, 16, \dots, (21)$



**8.** For any  $\epsilon > 0$ , and  $\delta > 0$ , there exist a step size  $h$  and a positive integer  $M$  such that

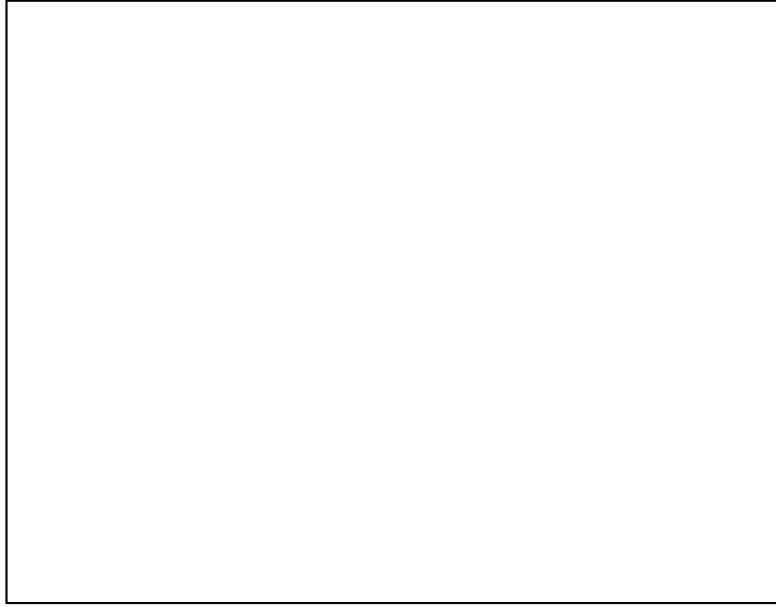
$$|e^{-xy} - G_e(x, y)| \leq \epsilon, \quad \text{for } xy \geq \delta, \quad (41)$$

where

$$G_e(x, y) = \frac{hx}{2\sqrt{y}} \sum_{j=0}^M e^{-x^2}$$







be an approximation of the kernel by Gaussians valid for  $\leq$



A.  $u(z, \cdot) \in C^2(\mathbb{R}^d)$ ,  $z > 0$ ,  $u_{zz} + \Delta u = 0$ ,  $u(0, \cdot) = u_0(\cdot)$ ,  $u(z, \cdot) \rightarrow 0$  as  $z \rightarrow \infty$ .

$$u(z, \cdot) = \int_{\mathbb{R}^d} \mathcal{P}(z, \cdot - \cdot) u_0(\cdot) d\cdot, \quad z \geq 0, \tag{56}$$

$$\mathcal{P}(z, \cdot) = \frac{2}{d} \frac{z}{(z^2 + \|\cdot\|^2)^{(d+1)/2}}$$

$$S_\infty(z^2 + \|\cdot\|^2) = \frac{zh}{(d+1)/2}$$



$$0 \leq n \leq 2N, \quad a > 0, \quad m = \frac{2N}{a} t_m, \quad [0, 1], \quad h(x) \quad (66)$$

$$\left| h(x) - \sum_{m=1}^M w_m e^{-mx} \right| < \dots \quad (67)$$

$$h \quad (66)$$

- B  $(N+1) \times (N+1)$   $0 \leq k \leq N$   $h_k = h_{k+1}$   $h_n = h(a \frac{n}{2N}), 0 \leq n \leq 2N.$
- $(u_0, \dots, u_N),$   $(16, \dots, 22);$   $M+1$   $M \approx 1$   $M = \mathcal{O}(N^{-1})$
- C  $m$   $u$

$$x = \frac{1}{2} - \frac{1}{2} \cos \theta, \quad \theta \in [0, \pi] \tag{70}$$

$$\tilde{t}^* = \frac{1}{2} - \frac{1}{2} \cos \theta^* \tag{70}$$

$> 1$ , ...

$$\frac{(\cdot, x)}{(\cdot)} \leq \frac{e}{(\cdot)} x^{-1} e^{-x} \leq \dots$$

$$\dots \text{ fi } \dots \tag{69}$$

$$x > d(e - 1) \tag{71}$$

$$d = e/(e - 1) \approx 1.582, \dots, 14, \dots, x \dots$$

$$x \geq d$$



A.3. Proof of Theorem 5

2.1.1  $t_0 = 0$  (20)  $h$  (15)  $t_*$   $t^*$  7.

$$\tilde{h} = \frac{10}{2^{-1} + 2^1}$$

$$\frac{\tilde{t}^* - \tilde{t}_*}{\tilde{h}}$$

$\tilde{t}^*, \tilde{t}_*$  (33) (34).

$$\tilde{t}^* - \tilde{t}_* = -1 + \frac{1}{-1 + \dots} -1 + \dots -1 + \dots \left( \frac{-1 + 1}{(1 + \dots)^1} \right) + \frac{1}{2}$$

$$\leq -1 + \frac{1}{-1 + \dots} -1 + \dots -1 + \frac{3}{2}$$

15. (23) (27) (28).

15. Let  $g(x) = \frac{(x+1)^{\frac{1}{x}}}{x+1}$  for  $x > 0$



33. ... B 0 ... 0 ...

34. ... C ... 121 (7) (2004) 2866 2876. ... B 0 ... 0 ... C ... 121 (14) (2004) 6680 6688.

35. ... 0 ... A ... C ... 20 (2) (1999) 699 718