Check that the functions are linearly independent

$$W\left(x^{3}; x\right) = \begin{vmatrix} x^{3} & x \\ 3x^{4} & 1 \end{vmatrix} = 4x^{3} \neq 0$$

so the two functions are linearly independent. $\left\{x^{3}; x\right\}$ is a basis for the solution space.

(c) We need to use variation of parameters so start by putting the differential equation into standard form

$$y'' + \frac{3}{x}y' \quad \frac{3}{x^2}y = \frac{1}{x^3}$$

We'll let $y_1 = x^{3}$ and $y_2 = x$, $f(x) = x^{3}$ from the differential equation and $W[y_1; y_2] = 4x^{3}$ from part (b). The particular solution will have the form $y_p = v_1y_1 + v_2y_2$ where

$$v_1' = \frac{y_2 f}{W[y_1; y_2]} = \frac{x\; x^{3}}{4x^{3}} = \frac{x}{4} \Rightarrow v_1 = \int \frac{x}{4}\,dx = \frac{x^2}{8}$$

$$v_2' = \frac{y_1 f}{W[y_1; y_2]} = \frac{x^{3}\; x^{3}}{4x^{3}} = x$$

(d) (3 pts) If $\ = 3$ and the mass of the object is 4, what is the value of the spring/restoring constant if the oscillator is critically damped?

SOLUTION:

(a) $x(0) = \ 1;\ \dot{x}(0) = 0$

(b) $\ = 0;\ \omega_0 > 0;\ F_0 \neq 0;\ \ = \omega_0$

(c) i. yes
 ii. infinitely many
 iii. yes

(d) Critically damped means $4\ ^2\ \ 4\omega_0^2 = 0 \Rightarrow 9\ \ \frac{k}{4} = 0 \Rightarrow k = 36$

5. [2360/041322 (12 pts)] Characteristic equations for certain constant coefficient linear homogeneous differential equations are given along with a forcing function, $f(t)$. Give the form of the particular solution you would use to solve the nonhomogeneous [with the given $f(t)$] differential equation from which the characteristic equation was derived using the Method of Undetermined Coefficients. Do not solve for the coefficients.

(a) (4 pts) $r(3r\ \ 1) = 0;\ f(t) = \ 3 + \sin t$

(b) (4 pts) $r^2 + 2r + 5 = [r\ \ (\ 1 + 2i)][r\ \ (\ 1\ \ 2i)]) = 0;\ f(t) = e^{\ t} + 5\cos 2t$

(c) (4 pts) $(r\ \ 3)^3(r + 2) = 0;\ f(t) = te^{3t} + 2e^{\ 2t}$

SOLUTION:

(a) Homogeneous solutions are $1; e^{t/3};\ y_p = At + B\cos t + C\sin t$

(b) Homogeneous solutions are $e^{\ t}\cos 2t; e^{\ t}\sin 2t;\ y_p = Ae^{\ t} + B\cos 2t + C\sin 2t$

(c) Homogeneous solutions are $e^{3t}; te^{3t}; t^2e^{3t}; e^{\ 2t};\ y_p = t^3(At + B)e^{3t} + Cte^{\ 2t}$