APPM 2350—Exam 2

Friday, June 24th 1pm-2:35pm 2022

This exam has 4 problems. Show all your work and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1 (30 pts)

Consider the function

$$f(x,y) = \frac{\sqrt{x}}{y}$$

(a) Graph the level curve of $f(x,y)$ that passes through the point $(0,2)$. Label the value of f along the curve.

(b) On the same graph as part (a) graph the level curve where $f(x,y) = 1$. Label the value of f along this curve.

(c) On the same graph as part (a), graph one level curve where $f(x,y) < 0$. Label the value of f along this curve.

(d) At the point $(1,1)$, give a vector that points in the direction in the domain where this function *decreases* fastest.

(e) Sketch the vector you found in part (d) starting at $(1,1)$ on your graph from part (a).

(f) Use a *2nd order (i.e. quadratic)* Taylor approximation centered at $(1,1)$ to approximate $\frac{\sqrt{1.8}}{1.5}$

You can leave your answer as an unsimplified sum and/or difference of terms.

Problem 2 (22 pts) The temperature (in degrees Farenheit) in a region in space is given by

$$T(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}xyz$$

A particle is moving in this region and its position at time t is given by

$$\vec{r}(t) = 2\cos(\ t)\mathbf{i} + e^{(9-t^2)}\mathbf{j} - 2t\mathbf{k}$$

where time is measured in seconds and distance in meters.

(a) Use the chain rule to determine how fast the temperature experienced by the particle is changing in degrees Farenheit *per second* at the point $(x,y,z) = (-2,1,-6)$.

(b) How fast is the temperature experienced by the particle changing in degrees Farenheit *per meter* at the point $(x,y,z) = (-2,1,-6)$? (i.e. find the rate of change of the temperature with respect to distance in the direction the particle is moving at the point $(x,y,z) = (-2,1,-6)$).

Problem 3 (28 pts)

The following parts are not related:

(a) Find and classify all critical points of

$$g(x,y) = x^4 + y^4 - 4xy$$

(b) An airplane moves in a trajectory given by

$$\vec{r}(t) = 4t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k} \qquad t \geq 0$$

Given this trajectory, it will intersect the following surface twice:

$$z = 2x + 2y - y^2 - 8$$

Determine the tangent plane to the surface at the location where the airplane intersects the surface for a second time. Give your answer in standard (i.e. linear) form.

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Problem 4 (20 pts)

A mother puts her child on an amusement park ride that takes the child along a path in the xy-plane described by the equation $x^2 - 2x = 4y - y^2$. While the child is on the ride, the mother stands at the location $(x; y) = (0; 0)$.

(a) Use Lagrange multipliers to find the minimum and maximum distances from the mother to the child during the ride.

(b) Give the $(x; y)$ coordinates of the child at the minimum and maximum distances.

End Of Exam
