

- i. [8 pts] Are there any points in the plate that are locally thicker or thinner than their nearby surroundings? If so, where are they? If not, explain why not.
- ii. [5 pts] Is there a thinnest part of the plate? Do not find it, simply answer YES or NO and give a brief explanation justifying your answer.

SOLUTION:

(a) We need the gradient of the magnetic field.

$$\nabla B = \frac{1}{xyz}(yz)\mathbf{i}; \frac{1}{xyz}(xz)\mathbf{j}; \frac{1}{xyz}(xy)\mathbf{k} = \frac{1}{x}\mathbf{i}; \frac{1}{y}\mathbf{j}; \frac{1}{z}\mathbf{k}$$

The maximum rate of change of the magnetic field occurs in the direction of the gradient so the ship should be aimed in the direction

$$\nabla B(1; 1; 2) = \left(1; 1; \frac{1}{2}\right) = \mathbf{i} + \mathbf{j} + \frac{1}{2}\mathbf{k}$$

and the maximum rate of change of the magnetic field will be given by

$$|\nabla B(1; 1; 2)| = \sqrt{1^2 + 1^2 + \left(\frac{1}{2}\right)^2} = \frac{3}{2}$$

(b) i. We need to find and classify the critical points.

$$h_x = 2xe^y = 0 \Rightarrow x = 0$$

$$h_y = e^y(2y + y^2 - x^2) = 0 \Rightarrow 2y + y^2 = 0 \quad (\text{since } x = 0) \Rightarrow y = 0; -2$$

Critical points are (0; 0); (0; -2). Now apply the Second Derivatives Test.

$$D(0; 0) = h_{xx}(0; 0)h_{yy}(0; 0) - [h_{xy}(0; 0)]^2 = (-2)(2) - 0^2 = -4 < 0 \Rightarrow (0; 0) \text{ is a saddle point}$$

$$D(0; -2) = h_{xx}(0; -2)h_{yy}(0; -2) - [h_{xy}(0; -2)]^2 = 2e^{-2} - 2e^{-2} - 0^2 = 4e^{-4} > 0$$

$$\text{and } h_{xx}(0; -2) = 2e^{-2} < 0 \Rightarrow h(0; -2) \text{ is a local maximum}$$

The thickness is a local maximum at (0; -2) so there is a point that is locally thicker than its nearby surroundings. There are no points in the plate that are locally thinner than their surroundings.

- ii. YES. The thickness is a continuous function and the plate is a closed, bounded region so the Extreme Value Theorem applies. Since the interior critical points are a saddle and a local maximum, the thinnest part of the plate will be on the boundary.

3. [2350/031523 (25 pts)] Let $g(x; y) = \cos(xy) +$

(b)

$$dg = \frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial y} dy = y \sin(xy) + y^2 dx + [x \sin(xy) + 2xy] dy$$

At (1; 1) we have $dg = 1 dx + 2 dy$ so that g is more sensitive to small changes in y .

(c) You arrive at the point $(\frac{1}{2}; 1)$ when $t = 4$ and the rate of change of temperature with respect to time is given by

$$\begin{aligned} \frac{dg}{dt} &= r g'(t) = h y \sin(xy) + y^2; \quad x \sin(xy) + 2xy \quad \frac{1}{2} t^{-3/2}; \quad \frac{t}{8} \\ \Rightarrow \frac{dg}{dt} &= h (1) \sin(=2) + 1^2; \quad (1=2) \sin(=2) + 2(1=2)(1) \quad \frac{1}{2} 4^{-3/2}; \quad \frac{4}{8} \\ &= \frac{D}{1}; \quad \frac{E}{2} \quad \frac{1}{16}; \quad \frac{1}{2} = \frac{1}{16} + \frac{1}{16} + \frac{8}{16} \quad \frac{4}{16} = \frac{1}{16} (7 - 3) \end{aligned}$$

Since this is negative, the temperature is decreasing at a rate of $\frac{1}{16}(7 - 3)$

4. [2350/031523 (16 pts)] A skateboard park has a new track in the form of the hyperbola $\frac{1}{4}x^2 - y^2 = 12$. You are standing at the point $(x; y) = (0; 10)$

SOLUTION:

(a)

$$f_{1; \frac{1}{2}} = 1$$

$$f_x(x; y) = 2xe^{(x^2+2y)} \Rightarrow f_x\left(1; \frac{1}{2}\right) = 2$$

$$f_{xx}(x; y) = 2e^{(x^2+2y)}(2x^2+1) \Rightarrow f_{xx}\left(1; \frac{1}{2}\right) = 2$$

$$f_{xy}(x; y) = 4xe^{(x^2+2y)} \Rightarrow f_{xy}\left(1; \frac{1}{2}\right) = 4$$