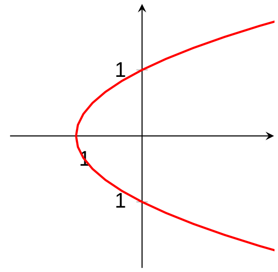
APPM 2350—Exam 1

Wednesday Sep 22nd, 6:30pm-8pm 2021

(c) Sketch.



(b) The missile will travel along the tangent line to the jet at the point $(2, 2, \frac{4}{3})$.

To find the equation of this tangent line, we need a point on the line and and a vector in the direction of the missile.

Point on the tangent line: $(2, 2, \frac{4}{3})$

Vector in the direction of the missile= $\mathbf{r}'(2) = \langle 1, 2, 2 \rangle$ (using our parameterization from part (a)).

However, notice that the speed of $\mathbf{r}'(2)$ is not the same as the missile's speed.

The missle's velocity vector = $(12)\frac{\mathbf{r}'(2)}{||\mathbf{r}'(2)||} = 12\frac{\langle 1,2,2\rangle}{3} = \langle 4, 8, 8 \rangle$

Thus, the missile will travel along the line given below for $t \geq 0$:

$$x = 2 + 4t$$

$$y = 2 + 8t$$

$$z = \frac{4}{3} + 8t$$

After traveling along this line for 5 seconds, the missile will be at the point:

$$x = 2 + 4(5) = 22$$

$$y = 2 + 8(5) = 42$$

$$z = \frac{4}{3} + 8(5) = \frac{124}{3}$$

$$\Rightarrow \quad (x, y, z) = \left(22, 42, \frac{124}{3}\right)$$

Problem 4 (20 points)

A particle travels along a curve parameterized by

$$\mathbf{r}(t) = \langle 4t, \cos(3t), \sin(3t) \rangle, \quad 0 \leq t \leq \pi/3$$

where t is time.

(a) At what coordinates (x, y, z), if any, is the particle's unit normal vector, $\mathbf{N}(t)$ parallel to the following plane? Explain/justify your answer.

$$\frac{3}{2}x + y - \sqrt{3}z = \frac{2}{3}$$

(b) At what time(s), if any, is the curvature of the particle's path equal to $\frac{1}{2}$? Explain/justify your answer.

<u>**SOLUTION:**</u>

(a)

$$\mathbf{r}'(t) = \langle 4, -3\sin(3t), 3\cos(3t) \rangle$$

$$||\mathbf{r}'(t)|| = \sqrt{16 + 9\sin^2(3t) + 9\cos^2(3t)} = 5$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} = \frac{1}{5}\langle 4, -3\sin(3t), 3\cos(3t) \rangle$$

$$\frac{d}{dt}\mathbf{T}(t) = \frac{1}{5}\langle 0, -9\cos(3t), -9\sin(3t) \rangle$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||} = \langle 0, -\cos(3t), -\sin(3t) \rangle$$

(d)

$$
\begin{aligned}
a_N' &= 0 \\
\left(\frac{2c}{\sqrt{1+4(t-1)^2}}\right)' &= 0 \\
-\frac{8c(t-1)}{(1+4(t-1))^{3/2}} &= 0 \\
t &= 1 \quad \text{(5 pts)}
\end{aligned}
$$

$a_N' > 0$ when $t \to 1^-$ and $a_N' < 0$ when $t \to 1^+$ $\Rightarrow$ $a_N(t = 1) = 2c$ is a maximum. (2 pts)

(e) $a_N(t = 1) = 2c = g$, therefore for the car not to roll over we need $c \leq \frac{g}{2}$