

1. (30 pts) Consider the two planes described by  $2y + z = 1$  and  $x + 2y + 2z = 3$ .

- (a) Is the point  $P(-1; 0; 1)$  on both planes, one of the planes, or neither?
- (b) Find the angle formed by the two planes.
- (c) Find equations for the line of intersection of these two planes. Express your answer in parametric and symmetric forms.
- (d) What is the shortest distance between the point  $Q(0; 1; 3)$  and the line of intersection?

**Solution:**

- (a) The point  $P(-1; 0; 1)$  satisfies both equations, so it lies on both planes.

$$0 + 1 = 1 \quad \text{and} \quad -1 + 0 + 2 = 3$$

- (b) The angle between the planes equals the angle between their normal vectors  $\mathbf{n}_1 = \langle 0; 2; 1 \rangle$  and  $\mathbf{n}_2 =$

thus

$$d = \frac{|\mathbf{PQ} \cdot \mathbf{v}|}{|\mathbf{v}|} = \frac{\rho \sqrt{29}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{\rho \sqrt{29}}{3}$$

### Alternate Solution

The shortest distance equals

$$\begin{aligned} |\mathbf{PQ} \cdot \text{proj}_{\mathbf{v}} \mathbf{PQ}| &= |\mathbf{PQ} \cdot \frac{\mathbf{PQ} \cdot \mathbf{v}}{|\mathbf{v}|^2} \mathbf{v}| \\ &= |h1; 1; 2i \cdot \frac{h1; 1; 2i \cdot h2; 1; 2i}{3^2} h2; 1; 2i| \\ &= |h1; 1; 2i \cdot \frac{5}{9} h2; 1; 2i| \\ &= |h \frac{1}{9}; \frac{14}{9}; \frac{8}{9}i| = \frac{\rho \sqrt{29}}{3} \end{aligned}$$

2. (16 pts) A particle is moving in the direction  $\mathbf{v} = \mathbf{i} + \mathbf{j}$  when a force of  $\mathbf{F} = 3\mathbf{j} + 4\mathbf{k}$  is applied to it.
- Decompose the vector  $\mathbf{F}$  into a sum of 2 vectors: one vector parallel to the particle's direction of motion and the other vector orthogonal.
  - Find a unit vector that is orthogonal to  $\mathbf{F}$ .

### Solution:

- (a) A vector parallel to the particle's direction of motion is the projection of  $\mathbf{F}$  onto  $\mathbf{v}$ .

$$\text{proj}_{\mathbf{v}} \mathbf{F} = \frac{\mathbf{v} \cdot \mathbf{F}}{|\mathbf{v}|^2} \mathbf{v} = \frac{h1; 1; 0i \cdot h0; 3; 4i}{\rho \sqrt{2}^2} h1; 1; 0i = \frac{3}{2} h1; 1; 0i = \frac{3}{2}; \frac{3}{2}; 0$$

A vector orthogonal to the projection is

$$\text{orth}_{\mathbf{v}} \mathbf{F} = \mathbf{F} - \text{proj}_{\mathbf{v}} \mathbf{F} = h0; 3; 4i - \frac{3}{2}; \frac{3}{2}; 0 = \frac{3}{2}; \frac{3}{2}; 4$$

Therefore

$$\mathbf{F} = h0; 3; 4i = \frac{3}{2}; \frac{3}{2}; 0 + \frac{3}{2}; \frac{3}{2}; 4$$

- (b) The vector  $\mathbf{F} = 3\mathbf{j} + 4\mathbf{k}$  lies in the  $yz$ -plane and has slope  $\frac{4}{3}$ . An orthogonal vector with slope  $\frac{4}{3}$



