1. Let R be the region bounded by $y = 2 - x$ and $y = x^2$

(a) (5 points) Sketch the region. Be sure to label all axes, curves, and intersection points.

SOLUTION:

Figure 1: Bounded Area Region

(b) (7 points) Set up the dx integral(s) (integral(s) with respect to x) which, if evaluated, would give the area of the region. DO NOT EVALUATE.

SOLUTION:

$$\text{Area} = \int_{-2}^{1} (2 - x - x^2)\,dx$$

(c) (7 points) Set up the dy integral(s) (integral(s) with respect to y) which, if evaluated, would give the area of the region. DO NOT EVALUATE.

SOLUTION:

$$A = \int_0^1 2\sqrt{y}\,dy + \int_1^4 (2 - y) - \sqrt{(y)}\,dy$$

(d) (5 points) Find the area of the region R from either (b) or (c).

SOLUTION:

Evaluation of either (b) or (c) will yield a result of $\frac{9}{2}$

2. Consider the curve $y = \sin(x)$.

(a) (8 points) Use the Midpoint Rule with $n = 4$ sub-intervals to estimate the integral of this curve from $x = 0$ to $x = \pi$.

SOLUTION:

Here, we know that $n = 4$; $a = 0$; and $b = \pi$. Thus, we can say $x = \frac{(b-a)}{n} = \frac{\pi}{4}$. We now divide the interval [0,4] into n

3. (a) (10 points) Evaluate the following integral $\displaystyle\int_0^3 \frac{1}{(x-1)^{2/3}}\,dx$ and determine whether it converges or diverges.

SOLUTION:

4. Evaluate the following integrals. Show all work!

(a) (12 points) $\int \frac{\ln(x)\ln(\ln(x))}{x}\,dx$

SOLUTION:

$u = \ln(x)$

$du = \frac{1}{x}dx$

$\Rightarrow \int u \cdot \ln(u)\,du \Rightarrow u = \ln(u)\,,\; du = \frac{1}{u}du; dv = u; v = \frac{u^2}{2}$

$\Rightarrow \frac{\ln(u)u^2}{2} - \int \frac{u}{2}du = \frac{u^2 \cdot \ln(u)}{2} - \frac{u^2}{4} + c$

$= \boxed{\frac{(\ln(x))^2 \cdot \ln(\ln(x))}{2} - \frac{(\ln(x))^2}{4} + c}$

(b) (12 points) $\int x^3\sqrt{4+x^2}\,dx$

SOLUTION:

$x = 2\tan(\theta); dx = 2\sec^2\theta d\theta$

$\Rightarrow \int 8\tan^3\theta\sqrt{4+4\tan^2\theta}\,2\sec^2\theta d\theta$

$\Rightarrow 32\int \tan^3\theta\sec^3\theta d\theta$

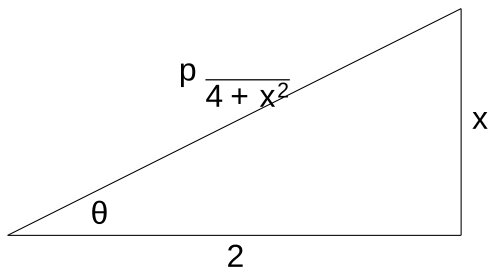
$= 32\int (\sec^2\theta - 1)\sec^2\theta\sec(\theta)\tan(\theta)d\theta$

$\Rightarrow u = \sec\theta; du = \sec\theta\tan\theta d\theta$

$= 32\int (u^4 - u^2)du$

$= 32[\frac{u^5}{5} - \frac{u^3}{3}] + c$

$= 32[\frac{\sec^5(\theta)}{5} - \frac{\sec^3(\theta)}{3}] + c$



$= \boxed{32\left(\frac{\left(\frac{\sqrt{4+x^2}}{2}\right)^5}{5} - \frac{\left(\frac{\sqrt{4+x^2}}{2}\right)^3}{3}\right) + c}$

(c) (12 points) $Z \quad 3x^2$