

1. (28 points) The following problems are not related.

(a) Find the general antiderivative of  $g(x) = \frac{e^{p\sqrt{x}}}{\sqrt{x}}$ .

(b) Use logarithmic differentiation to find the derivative of  $y = (x^4 + 1)^x$ . *You do not need to simplify your answer.*

(c) Find the derivative of  $f(x) = \int_0^{\cos(x)} \frac{1}{1+t^3} dt$ .

2. (26 points) The following problems are not related:

(a) Find the derivative of  $f(x) = \ln \tan^{-1}(x)$ .

(b) Evaluate the definite integral  $\int_0^{\ln(3)} \sinh(x) \cosh(x) dx$ , and fully simplify your answer.

(c) Determine the value of the limit  $\lim_{x \rightarrow 0^+} x^2 \ln(x^2)$ .

3. (16 points) Find the area of the largest rectangle which is symmetric around the  $y$ -axis, bounded below by the  $x$ -axis, and which has two corners touching the graph of  $f(x) = \frac{1}{1+x^2}$ . *Fully justify your answer by using an appropriate test.*



5. (12 points) For what value of  $a$  is the following function continuous?

$$f(x) = \begin{cases} 2x^2 + x + a; & x \leq 0 \\ \frac{x}{2\sin(x)}; & x > 0 \end{cases}$$

Justify your answer with appropriate computations.

6. (18 points) Consider the function

$$g(x) = \arctan(x) + \frac{1}{x^2 - 4}$$

(a) Find the domain of the function, and give your answer in interval notation.

(b) Find all horizontal asymptotes of  $g(x)$ , and justify your answer with limits.

7. (16 points) The half-life of the chemical element cobalt-56 is approximately 77 days. Suppose we have a 10 milligram sample of cobalt-56..

(a) Find a formula for the mass of cobalt-56 remaining after  $t$  days.

(b) How long will it take for only 1 milligram of cobalt-56 to remain in the sample? *It is OK for your answer to have a logarithm in it.*

8. (16 points) For each of the following questions, give a short justification for your answer.

(a) If  $f(x)$  is an odd function and  $\int_3^0 f(x) dx = 1$ , find  $\int_3^3 f(x) dx$ .

(a)

) is an odd function and