

1. (24 points) The following problems are not related. If a limit does not exist, you must say so. If you use a theorem, clearly state its name and show that its hypotheses are satisfied.

(Reminder: You may not use L'Hôpital's Rule or "Dominance of Powers" in any solutions on this exam.)

(a) $\lim_{x \rightarrow 0} \frac{\sec x}{4x \cot 2x}$

(b) $\lim_{x \rightarrow 1} \frac{\sin^2 x}{x}$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{5 - x^2}$

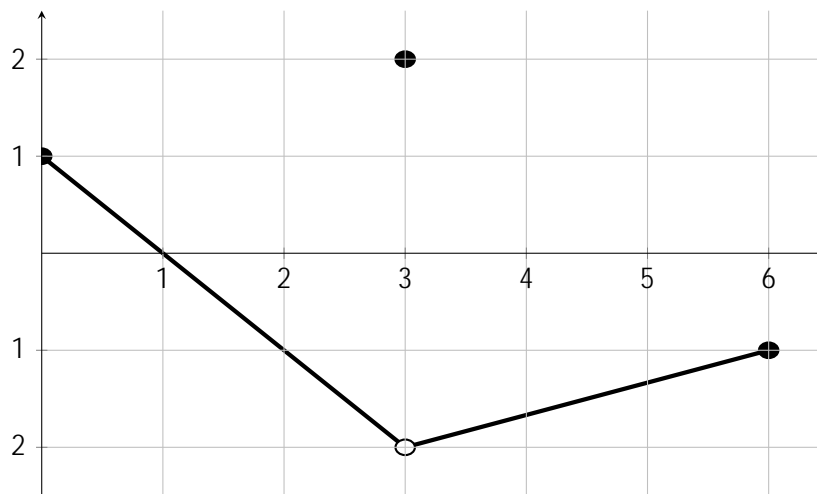
2. (21 points) The following problems are unrelated.

(a) Given that $\csc \theta = \sqrt{5}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, find the values of $\tan \theta$ and $\cos(2\theta)$.

(b) Find all values of x in the interval $[0; \pi]$ that satisfy $\tan x \sec x = 4 \sin x$.

(c) A squirrel is up a tree, and it sees a peanut on the ground some distance away. If the straight-line distance between the peanut and the squirrel is 50 ft, and the angle between the straight-line and the tree is $\theta = 6$ radians, how far down the tree and across the ground must the squirrel travel to reach the peanut? Give your answer with appropriate units.

3. (15 points) Shown below is a graph of $y = f(x)$, which consists of two line segments with a single removable discontinuity.



- (a) Find a formula for $f(x)$.
- (b) Sketch a graph of $y = f(x) + 1$. Label the intercepts, if any.
- (c) Suppose we use the precise definition of a limit to verify the value of $\lim_{x \rightarrow 5} f(x)$, and we find that if $4 < x < 6$, then $\frac{5}{3} < f(x) < 1$. What are the corresponding values of ϵ and δ ? (recall the precise definition of a limit: the limit of $f(x)$ as x approaches a is L if for every number $\epsilon > 0$, there is a corresponding $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.)

4. (20 points) Consider the function $g(x) = \frac{2x^2 - 12x + 16}{x^2 - 7x + 12}$.

(Reminder: You may not use L'Hôpital's Rule or "Dominance of Powers" in any solutions on this exam.)

- (a) Find the domain of $g(x)$. Express your answer in interval notation.
- (b) Find and classify all discontinuities of $g(x)$; justify your answers by calculating the appropriate limits.
- (c) Find the horizontal asymptotes, if any; justify your answers by calculating the appropriate limits.

5. (10 points) Consider the function

$$f(x) = \begin{cases} b \cos(x); & x < 1 \\ \frac{2x - 2}{3}; & x > 1 \end{cases}$$

Find the value of b such that $\lim_{x \rightarrow 1} f(x)$ exists. Justify your answer by calculating appropriate limits.

6. (10 points) Show that the equation $x^2 = \sin x \cos x$ has at least one real solution. Indicate the interval where a solution can be found.