1. (32 pts) The position function of a particle is given by $s(t) = 4\sqrt{t} - t$ on the interval $1 \le t \le 16$, where position is in meters and time is in seconds.

   (a) Determine the particle's velocity function $v(t)$. Include the correct unit of measurement.

   Solution:

   $s(t) = 4\sqrt{t} - t = 4t^{1/2} - t$

   $v(t) = s'(t) = 2t^{-1/2} - 1 = \boxed{\frac{2}{\sqrt{t}} - 1 \text{ m/s}}, \quad 1 < t < 16$

   (b) Determine the total distance traveled by the particle on the interval 1

(c) i. Verify that all hypotheses of the Mean Value Theorem are satisfied for the given position function $s(t) = 4\sqrt{t} - t$ on the interval $1 \le t \le 16$.

Solution:

$s(t)$ is continuous on $[1, 16]$ and differentiable on $(1, 16)$

ii. Use the Mean Value Theorem to determine all time values c on the interval $1 \le t \le 16$, if any, for which the instantaneous velocity of the particle equals the average velocity of the particle on that interval. Include the correct unit of measurement.

Solution:

The Mean Value Theorem states that since the hypotheses have been satisfied, there exists at least one number c on the interval $(1, 16)$ such that

$$s'(c) = \frac{s(16) - s(1)}{16 - 1} = \frac{0 - 3}{15} = -\frac{1}{5}$$

Therefore, using the result from part (a), we have

$$v(c) = \frac{2}{\sqrt{c}} - 1 = -\frac{1}{5}$$

$$\frac{2}{\sqrt{c}} = 4$$

2. (11 pts) Let v represent a person's walking speed, expressed in miles per hour, and let p represent the corresponding walking pace, expressed in minutes per mile. The pace can be expressed as the following function of speed:

$$p(v) = \frac{60}{v}; \quad v > 0$$

(a) Find the linearization $L(v)$ that approximates $p(v)$ near $v = 4$.

Solution:

$$p(v) \quad L(v) = p(4) + p^0(4)(v \quad 4)$$

$$p(4) = \frac{60}{4} = 15$$

$$p^0(v) = \quad \frac{60}{v^2} \quad) \quad p^0(4) = \quad \frac{60}{4^2} = \quad \frac{15}{4}$$

$$L(v) = \boxed{15 \quad \frac{15}{4}(v \quad 4)}$$

(b) Use your linearization from part (a) to estimate the walking pace of a person moving at 4.2 miles per hour. Include the correct unit of measurement. You must use linearization to earn credit.

Solution:

$$p(4{:}2) \quad L(4{:}2) = 15 \quad \frac{15}{4}(4{:}2 \quad 4) = 15 \quad \frac{15}{4} \; \frac{1}{}$$

3. (26 pts) Consider the function $f(x) = \sin x + \cos^2 x$ on the interval $[0; 2\ \ =3]$.

(a) Identify all critical numbers of f on the specified interval.

Solution:

Critical numbers are values of x in the domain of f such that $f^0(x) = 0$ or $f^0(x)$ does not exist. There are no critical numbers of the latter type for this function.

$f^0(x) = \cos x + 2\cos x \ (\ \ \sin x) = \cos x(1 \ \ \ 2\sin x) = 0$

$x = \ \ =2$ is the only value of x

5.

6. (15 pts) Determine $g'(x)$ for the function $g(x) = \sqrt{3x+1}$ by using the definition of derivative. You must obtain g' by evaluating an appropriate limit to earn credit.

Solution:

The definition of derivative indicates that $g'(x) = \lim_{h \to 0} \dfrac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \dfrac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h}$

7. (22 pts) Consider the function $h(x) = \dfrac{\sin x}{x(x \quad =2)}$.

(a) Find the $(x; y)$