

1. (30 pts) Determine  $\frac{dy}{dx}$  for each of the following.

(a)  $y = \sin^4(x^3)$

(b)  $x^2 + xy + y^3 = 4$

(c)  $y = \frac{2x^2 + 1}{x \cos x}$  After fully differentiating, do not algebraically simplify your answer any further.

**Solution:**

(a)

$$\begin{aligned} \frac{d}{dx} \sin^4(x^3) &= 4 \sin^3(x^3) \frac{d}{dx} \sin(x^3) = 4 \sin^3(x^3) \cos(x^3) \frac{d}{dx} x^3 \\ &= 4 \sin^3(x^3) \cos(x^3) (3x^2) = \boxed{12x^2 \sin^3(x^3) \cos(x^3)} \end{aligned}$$

(b)

$$\frac{d}{dx} x^2 + xy + y^3 = \frac{d}{dx} [4]$$

$$2x + xy' + y + 3y^2 y' = 0$$

$$y'(x + 3y^2) = -(2x + y)$$

$$y' = \boxed{\frac{-(2x + y)}{x + 3y^2}}$$

(c)

$$\begin{aligned} \frac{d}{dx} \frac{2x^2 + 1}{x \cos x} &= \frac{x \cos x \frac{d}{dx} [2x^2 + 1] - (2x^2 + 1) \frac{d}{dx} [x \cos x]}{(x \cos x)^2} \\ &= \frac{x \cos x (4x) - (2x^2 + 1) (x \frac{d}{dx} [\cos x] + \cos x \frac{d}{dx} [x])}{(x \cos x)^2} \\ &= \boxed{\frac{x \cos x (4x) - (2x^2 + 1)(x \sin x + \cos x)}{(x \cos x)^2}} \end{aligned}$$

2. (25 pts) Parts (a) and (b) are unrelated.

(a) The position function of Particle P is given by  $s(t) = 2t + t^2$ , where  $s$  is in meters,  $t$  is in seconds, and  $t \geq 1$ .

i. Find the particle's velocity function  $v(t)$ . Include the correct unit of measurement.

3. (23 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve  $y = x^{3-2} - x^{1-2}$  at  $x = 4$ .

(b) Find all values of  $x$  on the interval  $[0; \pi]$  for which the curve  $y = \sin^2 x - \sin x$  has a horizontal tangent line.

**Solution:**

$$(a) y'(x) = \frac{3}{2}x^{1-2} - \frac{1}{2}x^{-1-2} = x^{-1-2} - \frac{3}{2}x^{-1-2} = \frac{x^{-1-2}}{2} (2 - 3) = \frac{3x^{-1-2}}{2}$$

$$y'(4) = \frac{11}{4}$$

$$y(4) = 4^{3-2} - 4^{1-2} = 8 - 2 = 6$$

Tangent line:  $y - 6 = \frac{11}{4}(x - 4)$

Normal line:  $y - 6 = \frac{4}{11}(x - 4)$

$$(b) y'(x) = 2 \sin x \cos x - \cos x = \cos x(2 \sin x - 1) = 0$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$$

$$2 \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}; \frac{5\pi}{6}$$

Therefore,  $x = \frac{\pi}{6}; \frac{\pi}{2}; \frac{5\pi}{6}$

4. (22 pts) Parts (a) and (b) are unrelated.

(a) Determine  $f'(x)$  for the function  $f(x) = \frac{1}{x+1}$  by using the **definition of derivative**.

You must obtain  $f'$  by evaluating the appropriate **limit**

In order for  $g$  to be differentiable at  $x = 2$ ,  $g$  must also be continuous at  $x = 2$ .

$\lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2^+} g(x) = g(2)$  leads to the following:

$$\frac{3}{8} 2^3 = (2^2) + \frac{17}{2}(2) + c$$

$$3 = 4 + 17 + c$$

$$c = \boxed{10}$$