

1. (30 pts) Determine  $\frac{dy}{dx}$  for each of the following.

(a)  $y = \frac{\sin x}{2x + 1}$

**Solution:**

$$\frac{dy}{dx} = \frac{(2x + 1) \frac{d}{dx}[\sin x] - \sin x \frac{d}{dx}[2x + 1]}{(2x + 1)^2} = \boxed{\frac{(2x + 1) \cos x - 2 \sin x}{(2x + 1)^2}}$$

(b)  $x^3 - y^3 = 5xy$

**Solution:**

$$\frac{d}{dx}[x^3 - y^3] = \frac{d}{dx}[5xy]$$

$$3x^2 - 3y^2 y' = 5(xy' + y)$$

$$3x^2 - 5y = (5x + 3y^2)y'$$

$$y' = \frac{dy}{dx} = \boxed{\frac{3x^2 - 5y}{5x + 3y^2}}$$

(c)  $y = 4 \cos^5(2x)$

**Solution:**

$$y = 4 \cos^5(2x) = 4[\cos(2x)]^5$$

$$\frac{dy}{dx} = (4)(5)[\cos(2x)]^4 \frac{d}{dx}[\cos(2x)] = 20 \cos^4(2x) \sin(2x) \frac{d}{dx}[2x]$$

$$\frac{dy}{dx} = \boxed{40 \cos^4(2x) \sin(2x)}$$

2. (25 pts) The position value of a particle is given by  $s(t) = t^2 - 4t^{1.5} + 4t$ , where  $t \geq 0$  is in seconds and position is in feet. For each of the following, be sure to include the correct unit of measurement.

(a) Find the particle's velocity function  $v(t)$ .

**Solution:**

$$v(t) = s'(t) = \frac{d}{dt} (t^2 - 4t^{1.5} + 4t) = 2t - (4)(1.5)t^{0.5} + 4 = \boxed{2t - 6t^{0.5} + 4 \text{ ft/s}}$$

(b) Determine the particle's speed at  $t = \frac{9}{4}$  seconds.

**Solution:**

$$v\left(\frac{9}{4}\right) = \left(2\right)\left(\frac{9}{4}\right) - (6)\left(\frac{9}{4}\right)^{0.5} + 4 = \frac{9}{2} - (6)\left(\frac{3}{2}\right) + 4 = \frac{9}{2} - \frac{18}{2} + \frac{8}{2} = \frac{1}{2} = \boxed{\frac{1}{2} \text{ ft/s}}$$

(c) Find the particle's acceleration function  $a(t)$ .

**Solution:**

$$a(t) = v'(t) = \frac{d}{dt} (2t - 6t^{0.5} + 4) = 2 - (6)(0.5)t^{-0.5} = \boxed{2 - 3t^{-0.5} \text{ ft/s}^2}$$

3. (25 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve  $y = x^3 - 2x^2 + x + 10$  at  $x = -1$ .

**Solution:**

$$y(-1) = (-1)^3 - 2(-1)^2 + (-1) + 10 = -1 - 2 - 1 + 10 = 6$$

The point of tangency is  $(-1; 6)$ .

4. (20 pts) Parts (a) and (b) are unrelated.

(a) Determine  $f'(x)$  for the function  $f(x) = \frac{p}{x+1}$  by using the **definition of derivative**. (You must obtain  $f'$  by evaluating the appropriate limit to earn credit.)

**Solution:**

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{p}{(x+h)+1} - \frac{p}{x+1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{p}{x+h+1} - \frac{p}{x+1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{p}{x+h+1} + \frac{p}{x+1}}{\frac{p}{x+h+1} + \frac{p}{x+1}} \cdot \frac{h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h+1) - (x+1)}{h \left( \frac{p}{x+h+1} + \frac{p}{x+1} \right)} = \lim_{h \rightarrow 0} \frac{h}{h \left( \frac{p}{x+h+1} + \frac{p}{x+1} \right)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\frac{p}{x+h+1} + \frac{p}{x+1}} = \frac{1}{\frac{p}{x+0+1} + \frac{p}{x+1}} = \boxed{\frac{1}{2p(x+1)}}
 \end{aligned}$$

(b)  $\lim_{x \rightarrow 1} \frac{x^8 + 2x^5 - 3}{x - 1}$  represents the derivative of a certain function  $f$  at a certain number  $a$ .

i. Identify  $f$  and  $a$ .

**Solution:**

The definition of a derivative states that  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ .

Since the given expression represents  $f'(a)$  for some  $f$  and  $a$ , we have

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow 1} \frac{x^8 + 2x^5 - 3}{x - 1}$$

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