1. The following are unrelated: (15 pts)

   (a) Rewrite each of the following without the absolute value symbol:

       i. $|2\pi - 6|$

   Solution:

   Since $\pi > 3$, $2\pi > 6$, $2\pi - 6 > 0$ so $|2\pi - 6| = \boxed{2\pi - 6}$

       ii. $|\sqrt{2} - 2|$

   Solution:

   Since $\sqrt{2} < 2$, $\sqrt{2} - 2 < 0$ hence $|\sqrt{2} - 2| = \boxed{-\left(\sqrt{2} - 2\right)}$ or $2 - \sqrt{2}$

(d) Add and simplify: $\dfrac{2}{\frac{9}{7}} + \dfrac{5}{12} + 7^0$

Solution:

$$\frac{2}{\frac{9}{7}} + \frac{5}{12} + 7^0 = \frac{14}{9} + \frac{5}{12} + 1 \tag{4}$$

$$= \frac{56}{36} + \frac{15}{36} + \frac{36}{36} \tag{5}$$

$$= \boxed{\frac{107}{36}} \tag{6}$$

(e) Simplify: $\dfrac{j^7 \, 3j + j2j}{2j \, 4j}$

Solution:

$$\frac{j^7 \, 3j + j2j}{2j \, 4j} = \frac{10 + 2}{8} \tag{7}$$

$$= \frac{12}{8} \tag{8}$$

$$= \boxed{\frac{3}{2}} \tag{9}$$

2. The following are unrelated. Leave your answers without negative exponents. (20 pts)

(a) $(\,5b^3)^2 7a^{\,3} a^6$

Solution:

$$(\,5b^3)^2 7a^{\,3} a^6 = (\,5)^2 (b^3)^2 7a^3 \tag{10}$$

$$= 25 b^6 7 a^3 \tag{11}$$

$$= \boxed{175 b^6 a^3} \tag{12}$$

(b) Simplify: $\dfrac{p\sqrt{\frac{32x^2}{2}}}{p\sqrt{\frac{p}{16}}}$

Solution:

$$\frac{p\sqrt{\frac{32x^2}{2}}}{p\sqrt{\frac{p}{16}}} = \frac{jx\sqrt{\frac{p}{2}\,16}}{p\sqrt{\frac{p}{4}}} \tag{13}$$

$$= \frac{jxj4\sqrt{\frac{p}{2}}}{2\sqrt{\frac{p}{2}}} \tag{14}$$

$$= \boxed{2jxj} \tag{15}$$

(c) Simplify: $\dfrac{2(x^{\,2}y^3)^3}{8x^{\,3}y^{\,1=3}}$

Solution:

$$\frac{2(x^{\,2}y^3)^3}{8x^{\,3}y^{\,1=3}} = \frac{2x^{\,6}y^9}{8x^{\,3}y^{\,1=3}} \tag{16}$$

$$= \frac{x^{\,3}y^{\frac{27}{3}}}{4y^{\frac{1}{3}}} \tag{17}$$

$$= \boxed{\frac{y^{\frac{28}{3}}}{4x^3}} \tag{18}$$

(d) Multiply to rewrite as a polynomial: $\left(x-1+3\right)\left(x-1-3\right)$

Solution:

$$\left(x-1+3\right)\left(x-1-3\right) = \left(x-1\right)^2 - 3^2 \tag{19}$$
$$= x-1-9 \tag{20}$$
$$= \boxed{x-10} \tag{21}$$

3. The following are unrelated: (12 pts)

(a) Find the domain of the expression (give your answer in interval notation): $\dfrac{\sqrt{x^2-9}}{x(x-3)}$

Solution: $x-3 = 0$ results in division by zero so $x = 3$ must be excluded from the domain. As well $x > 0$ since the square root of a negative number does not exist. So the answer is: $\boxed{(0,3) \cup (1)}$

(b) Combine into a single fraction: $\dfrac{1}{x^2-x-2} - \dfrac{3}{x+1}$

Solution:

$$\frac{1}{x^2-x-2} - \frac{3}{x+1} = \frac{1}{(x-2)(x+1)} - \frac{3}{x+1} \tag{22}$$
$$= \frac{1-3(x-2)}{(x-2)(x+1)} \tag{23}$$
$$= \boxed{\frac{7-3x}{(x-2)(x+1)}} \tag{24}$$

(c) Evaluate the expression: $\dfrac{1}{2}x^2 - x^{-1}$ when $x = -4$

Solution: $\dfrac{1}{2}(-4)^2 - \dfrac{1}{-4} = 8 + \dfrac{1}{4} = \dfrac{32}{4} + \dfrac{1}{4} = \boxed{\dfrac{31}{4}}$

4. The following are unrelated: (12 pts)

(a) Simplify: $\dfrac{3x^2 + 12}{x^2 - 5x}$

(b) Simplify the compound fraction: $\dfrac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1}$

Solution:

$$\frac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1} = \frac{\frac{3}{x^2} - \frac{x}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} \tag{27}$$

$$= \frac{3 - x}{9 - x^2} \tag{28}$$

$$= \frac{3 - x}{(3 - x)(3 + x)} \tag{29}$$

$$= \boxed{\frac{1}{3 + x}} \tag{30}$$

(c) Factor by grouping: $9x^3 - 18x^2 - 4x + 8$

Solution:

$$9x^3 - 18x^2 - 4x + 8 = 9x^2(x - 2) - 4(x - 2) \tag{31}$$

$$= (x - 2)(9x^2 - 4) \tag{32}$$

$$= (x - 2)\left[(3x)^2 - 2^2\right] \tag{33}$$

$$= \boxed{(x - 2)(3x + 2)(3x - 2)} \tag{34}$$

5. Is $x = -2$ a solution of the inequality $x^3 - 2x < 2$

7. Solve each of the following equations. If there are no solutions write NO SOLUTIONS: (10 pts)

(a) $\sqrt{8-y}+2 = y-4$

Solution:

$$\sqrt{8-y}+2 = y-4 \tag{40}$$
$$\sqrt{8-y} = y-6 \tag{41}$$
$$8-y = y^2-12y+36 \tag{42}$$
$$y^2-11y+28 = 0 \tag{43}$$
$$(y-7)(y-4) = 0 \tag{44}$$
$$y = 4, 7 \tag{45}$$

Plugging into the original equation, we find $y=4$ to be extraneous. Hence $\boxed{y=7}$

(b) Solve for h: $P = A + hdg$

Solution:

$$P = A + hdg \tag{46}$$
$$hdg = P - A \tag{47}$$
$$h = \boxed{\dfrac{P-A}{dg}} \tag{48}$$

8. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (8 pts)

(a) $3x + 1 < 6$

Solution:

$$3x + 1 < 6 \tag{49}$$
$$3x < 5 \tag{50}$$
$$x > \frac{5}{3} \tag{51}$$

Hence the interval of solution is $\boxed{\dfrac{5}{3}; 1}$
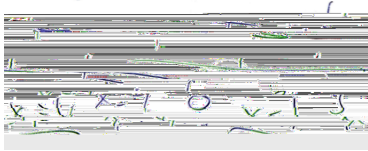
(b) $x^3 - 3x^2 \le 0$

Solution:
We start by factoring the left hand side, and then make use of a number line/sign chart to choose the relevant interval of solution

$$x^3 - 3x^2 \le 0 \tag{52}$$
$$x^2(x - 3) \le 0 \tag{53}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = 0$ and $x = 3$. Placing these on a number line and picking test values we obtain the following chart

Notice that $x = 0$ is a solution. Hence the solution is $\boxed{0 \cup [3; 7)}$.

9. Find all the solutions to the following equation, including the complex solutions (Hint: factoring will be important) $z^3 = 1$. (5 pts)

Solution:

$$z^3 - 1 = 0 \tag{54}$$
$$(z - 1)(z^2 + z + 1) = 0 \tag{55}$$

From which we conclude that $\boxed{z = 1}$ or

$$z = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \tag{56}$$
$$= \frac{-1 \pm \sqrt{-3}}{2} \tag{57}$$
$$= \boxed{\frac{-1 \pm \sqrt{3}i}{2}} \tag{58}$$