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An approximate renormalization for the break-up of invariant tori with three frequencies

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Abstract

Renormalization theory provides a description of the destruction of invariant tori for Hamiltonian systems of $1\frac{1}{2}$ or 2 degrees

dent resonances for ω, then $\omega = p$ where p is integral (remember the length of ω is unimportant). A frequency ω is *Diophantine* if there is a $K \neq 0$ and $\tau > 2$

such that $\forall\, m \in Z^3 \backslash 0$, $|m\,\omega|/|\omega| > K/|m|^{\tau}$.

When $A=B=C=0$, the momenta (u, v) are constant in time and every orbit lies on a three torus. If $\omega(u, v)$ is incommensurate, the orbit densely covers the torus. If ω is Diophantine, then the KAM theorem implies that there is a torus with this frequency for small values of the amplitudes. We are interested in determining the parameters for which such a torus

tersection of each pair of resonances defines rational frequencies $p_1 = [1, 0, 0]$, $p_2 = [0, 1, 0]$, $p_3 = [0, 0, 1]$. The frequencies p_i also delineate the cone: it is the

convex hull of the three vectors. We denote the cone by either of the matrices

$$M = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}, \quad P = (p_1, p_2, p_3)\,.$$

We assume ω is inside the cone, i.e. $\omega_i \geqslant 0$.

[19] R.S MacKay and J Stark, Phys Lett A 138 (1989) 113

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