ELSEVIER

156 (2001) 201–218

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www.elsevier.com/locate/physd

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28 A 2000; 27 2001; 17 A 2001

Ab t act

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MSC: 37 40; 37 40; 37 50

Keywords: ; A ;

1. I t ct

A (A): 2 2n A n $n > 1$, z, $z_n \to z$, U z z_n U z U. A 1964 [1] A A

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0167-2789/01/$ 2001

0167-2789(01)00282-2

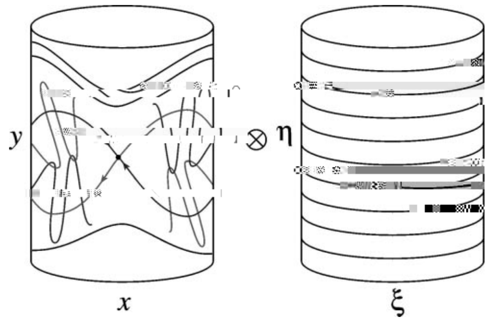

Fig. 1. ... (1) ... $h = 0$ ... (x, y), ... (ξ, η).

... $T^2 \times R^2$ ...

$$x' = x + y', \qquad \xi' = \xi + \eta', \qquad y' = y - k(1 + h \ldots \xi) \ldots x, \qquad \eta' = \eta - kh($$

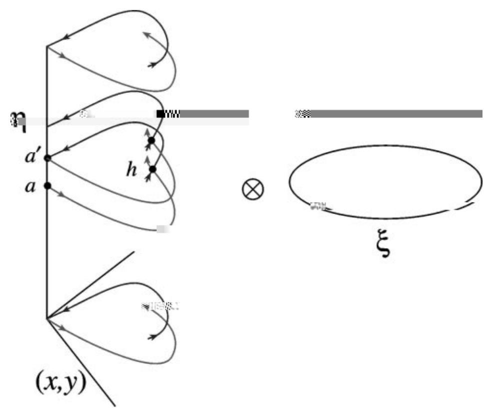

Fig. 2. [illegible] (1) [illegible] $h \neq 0$. [illegible] $\mathcal{C}$ [illegible] ξ, [illegible] C_a [illegible] C_a [illegible] $C_{a'}$ [illegible] h.

[illegible] a' [illegible] a. [illegible] [11] [illegible] [5,7].

[illegible] $h = 0$ [illegible] k [illegible]

… $(x, y) \mapsto (x', y')$, …

$$x' = x + \frac{1}{\varepsilon} y', \qquad y' = y + \quad V(x). \tag{4}$$

… $n = 1$, … $V(x) = k$ … (x), … [13].

3. A t- t ab t

A … [3,4,15], … (3) …

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4. C[illegible] t[illegible]

[illegible]

that C *is not identically zero. Then given any* $a < b$, *there is a nonzero measure of initial states* (ξ_0, ξ_1) *and a sequence* $c_t \in .(V)_+ \cup .(V)_-$ *such that the solution of* (14) *has momenta,* $\eta_t = T_2(\xi_{t-1}, \xi_t)$ *satisfying* $\eta_0 < a$ *and* $\eta_T > b$ *for some time T.*

P ... $c_- \in .(V)_-$ ... $c_+ \in .(V)_+$ ... ξ_t, ... $x_t = c_\pm$ ... $(\ C(\xi_t)) = \pm 1$.
... (14) ...

$$\tilde{L}(\xi, \xi') = T(\xi, \xi') + W(\xi) + \tilde{C}(\xi),$$

... $\tilde{C}$ ...

$$\tilde{C} = V(c_\pm(\xi)) \quad C(\xi) \geq 0.$$

... $\tilde{C}(\xi + 2\pi) - \tilde{C}(\xi) > 0$, ... $= \tilde{L}(\xi + 2\pi, \xi' + 2\pi) - \tilde{L}(\xi, \xi')$ ...
... 5, ... □

4.2. Standard example

...

$$L(x, x', \xi, \xi') = \tfrac{1}{2}\varepsilon(x' - x)^2 + \tfrac{1}{2}(\xi' - \xi)^2 + k \quad x(1 + h \quad \xi), \tag{15}$$

... $k > 0$ ... $h > 0$ ... (1), ... (15) ...
h

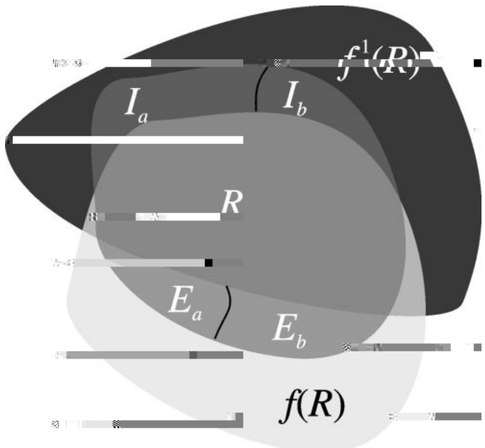
$\xi_t \to \pi$, $\eta^* = 2\pi m$. $(0, 2\pi m)$ $(\pi, 2\pi m)$, (16). $\eta^* = (2m+1)\pi$, $(0, \eta^*) \mapsto (\pi, \eta^*)$. (16) ( ). (16)

(16), g^*, g g^* g. (16) 2π $\mathrm{T}^2 = \{(\xi, \eta) : 0 < \xi, \eta < 2\pi\}$.

$$\eta^* = (\eta_{t+1} - \eta_t)^* = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} (U'(\xi_t)).$$

$$\langle \eta^* \rangle = \frac{1}{4\pi^2} \int_{\mathrm{T}^2} U'(\xi) = \frac{}{2\pi}.$$

(16) .012



4. ... R.

$$E = \{z \in R : f(z) \notin R\} = R \setminus f^{-1}(R).$$

... f ... μ, ... R ...:

$$\mu(E)=\mu(R \setminus f^{-1}(R))=\mu(R) - \mu(R \cap f^{-1}(R)) = \mu(R) - \mu(f(R) \cap R) = \mu(R \setminus f(R)) = \mu(I). \quad (17)$$

... S^t ... t. ... I ... *stays* ... R ... t. ...

$$S^0 = I, \qquad S^t = f(S^{t-1}) \cap R = f(S^{t-1} \setminus E).$$

... S^t ... $f^{-j}(S^t)$ ... R ... $j = 0, \ldots, t$, ... $(t+1)^{st}$, ... S^t ... R, ... $\sum_{t=0}^{\infty} \mu(S^t) < \mu(R)$. ... $\mu(S^t) \to 0$ ... $t \to \infty$. ... I ... R ... E.

E_b

$$\mu(p(I_a)\cap E_b)=\mu(p(I_a))-\mu(p(I_a)\cap E_a)\geq \mu(I_a)-\mu(E_a).\qquad\square$$

f_t … R … t

$$I_t=R\setminus f_t(R),\qquad E_t=R\setminus f_t^{-1}(R).$$

f_t … (17), … R … k … t … S_k^t.

$$S_k^k=I_{k-1},\qquad S_k^{t+1}=f_t(S_k^t\setminus E_t).$$

S_k^t … t … $k\leq t$ … R …

$$\sum_{k=-\infty}^{t}\mu(S_k^t)<\mu(R) \tag{18}$$

t.

Lemma 4. *Let f_t be a sequence of measure-preserving homeomorphisms, and R a measurable set with incoming sets I_t and exit sets* E_t …

5.2. Maps of the cylinder

[illegible] C, [illegible] C [illegible] $f(C)$

$$= \int_C y'\,x' - y\,x.$$

[illegible] C. [illegible] C [illegible] T [illegible] B. [illegible] $U \subset T$ [illegible]

$$U = \{z \in T : f^{-1}(z) \in B\}.$$

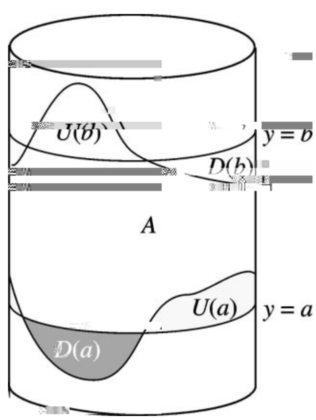
[illegible] $D \subset B$ [illegible]:

$$D = \{z \in B : f^{-1}(z) \in T\}.$$

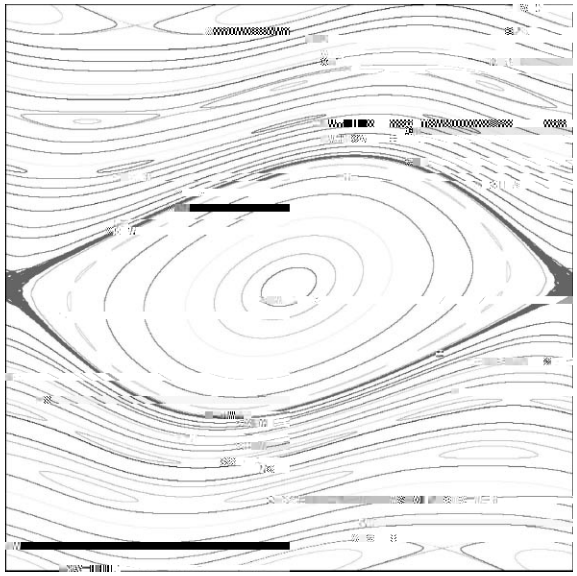
[illegible] $\mu(U) - \mu(D) =$ [illegible] 4 [illegible]

C[illegible] 5. *Suppose that f_t is a sequence of area and end-preserving homeomorphisms of the cylinder, and that the net flux $_t \geq \delta > 0$. Let A denote the annulus bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Then, there is a set of positive measure of orbits that cross the annulus.*

P[illegible] $U_t(a)$ [illegible] $D_t(a)$ [illegible] $\{y = a\}$ [illegible] f_t, [illegible] b, [illegible] 5. [illegible] A [illegible] $I_t = U_t(a) \cup D_t(b)$, [illegible] $E_t = f_t^{-1}(D_t(a)) \cup f_t^{-1}(U_t(b))$. [illegible] $\mu(U_t(b)) \geq \delta > 0$, [illegible] 4, [illegible] a [illegible] b [illegible] $f_t^{-1}(U_t(b))$. □



[illegible] 5. [illegible]



. 6. $k = 0.5$.

5.3. Standard map with net flux

(4) $(0, 2\pi) \times \mathbb{R}$. $= V(2\pi) - V(0)$. V :

$$x' = x + y', \qquad y' = y - k \quad (x) + \frac{1}{2\pi} \quad .$$

$= 0,$ $k < k_{cr} \approx 0.971635406$ $k = 0.5$. 6. 5, $\neq 0$ $y = 0$ $y = 2\pi$. $m, f(x, y + 2\pi m) = f(x, y) + 2\pi(m, m)$. . 7,

$y = 0$

$$x = \quad \overline{2\pi k}$$

$= 2\pi k$.

6. P t c t

(12) ε. $z_{t-1} = (x_{t-1}, x_t, \xi_{t-1}, \xi_t)$

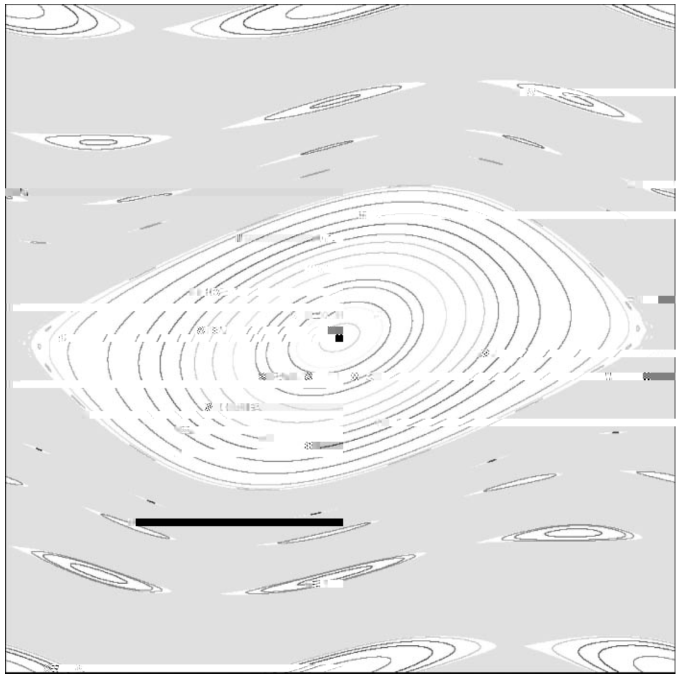


Fig. 7. … $k = 0.5$, … $\mathcal{F} = 4\pi^2/1000$. …).

$$z_t = \phi_\varepsilon(z_{t-1}) = \begin{pmatrix} x_t \\ -x_{t-1} + 2x_t + \frac{1}{\varepsilon} \; V(x_t) \; 1 + C(\xi_t) \\ \xi_t \\ -\xi_{t-1} + 2\xi_t + \;\; W(\xi_t) + V(x_t) \;\; C(\xi_t) \end{pmatrix}. \tag{19}$$

… $\varepsilon = 0$, … $x_t = c_t \in$ …(V) … ϕ_0. … $\varepsilon > 0$, … x- … .

L a 6. *Suppose that ϕ_ε, given by* (19), *is a C^2 map of T^4, such that $1 + C(\xi) \geq \tau > 0$. Then, for any sequence $\{c_0, c_1, \dots\}$ with $c_t \in$ …$(V) \cap \mathcal{A}$, any initial condition (ξ_0, ξ_1), and any $\delta > 0$, there exists an orbit $z_t = (x_t, x_{t+1}, \xi_t, \xi_{t+1})$, $t \geq 0$ of ϕ_ε such that*

$$|x_t - c_t| \leq \delta \quad \dots \quad t \geq 0,$$

provided

$$0 \leq \varepsilon < \varepsilon_0 = \frac{\sigma\tau}{(4\delta + a)}, \tag{20}$$

where $\sigma(\delta, b) \equiv \dots_{t\geq 0}| \; V(c_t \pm \delta)|$.

a $1 + C(\xi) \geq \tau$

$(1/\varepsilon)$ $V(c^{+}_{t+1})\tau > (4\delta + a)$ (ε) $\sigma\tau/\varepsilon > 4\delta + a$ (20).

W_t B_{t+1} W_{t+1}.

S W_0 $S = \mathrm{R}^2 \times (\xi_0, \xi_1) \cap W_0$.

z_0 S T S

U_t W_t $t \geq 1$. B

S

W_0, T B

T B W_t

S /(63.7 (' 8089 31 8 1 17.0455

$|\lambda_t^i| \le r^t$, $r > 1$ $r^2 - wr - 1 = 0$, $w =$ $_x(2 + |$ $W(x)|)$.

$$|\lambda_t| \le \tfrac{1}{2} M \delta^2 r^{2t}.$$

$W = 0$, λ_t t^2 T γ δ $|\lambda_t| \le \gamma$ $t \le T$. □

R a ξ_t $C(\xi)$,

6.1. Standard example, continued

7 (15), $V(x) = k$ x, $\delta < \pi/2$, $\sigma = k$ δ. $C(\xi) = h$ ξ, $\tau = 1 - h$, $h < 1$. $c_t \in \{0, \pi\}$, (x, y) $a = 2\pi$. 6

$$\varepsilon \le \varepsilon_0 = \frac{k(1-h) \quad \delta}{4\delta + 2\pi}.$$

7 $M = kh$ $W = 1$

$h < 1$ $\varepsilon \le \varepsilon_0$. ε 9.

7. C c

[illegible] [7] [illegible] [6,9,20] [illegible] [19] [illegible]

[illegible] [16] [illegible] (x, y) [illegible] [17].

Ac t

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